2nd Workshop on "Population and the Environment:

Modelling and Simulating this Complex Interaction"

18-19 May 2001, Rostock, Germany

A Model of Demographic and Economic Processes Interaction

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Workshop paper

May 15, 2001

Abstract

This paper gives a description of fertility influence on education level of population and its well-being. We carry out the consideration by a demographic model keeping age structure. We suppose that (i) a part of GDP is used for investment in education, (ii) the production level depends on the education level, and (iii) the pollution is directly proportional to the size of population. We are carrying on research on the dependence of average consumption on demographic structure of the population for an industrialised economy. In the model each age group of population has different skilled level determined by education and different level of potential labour productivity. The education level is completely determined by education expenditures. We have found the relation between the value of average consumption of population and the children-adults ratio, share of education expenditures in GDP. We have found that the quality of life can be improved by decrease of the population size not only ecologically but also economically.

Keywords**:** education level, skilled labour, fertility, well-being, age group, pollution, ecology, industrialised economy, quality of life, population size

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* I am grateful to the Russian Foundation of Basic Research (Grant #00-15-96118) and the Russian Humanitarian Scientific Foundation (Grant #01-02-00019a) for their financial support.

1. Introduction

We don't know the exact estimations of a maximum size of the earth population that is compatible with stability of biosphere, with stability of it components. But if the maximum size exists and it is overdrawn than we need in reduction of the population to this maximum size and then we need in its conservation at this level.

We investigate a dependence of per capita consumption on demographic structure of the population for an industrialised economy. In so doing we consider such age distributions of the population for which the age ratio is kept. Under the name "industrialised economy" we imply the economy for which the output is completely determined by labour employed in production and labour productivity, output norm on one unit of labour. An educational level of each age group of the population determines age labour productivity. The educational level is completely determined by expenses for education. The expenses for education are proportional to a gross domestic product.

2. Age dynamics of the population

Let $l(a, t)$ is an age distribution of the population on age a at time t. The rate of change for this value is defined by equation

$$\frac{\partial l}{\partial a} + \frac{\partial l}{\partial t} = -\mu l(a,t)\,, \tag{1}$$

where μ is a mortality rate, which is, generally speaking, age- and time-dependent, $\mu = \mu(a, t)$. For the description of age dynamics of the population it is necessary to set age distribution of the population at the initial time $t = 0$,

$$l(a, 0) = l_0(a), \tag{2}$$

and it is necessary to set the birth equation

$$l(0, t) = \int_0^{A_N} \beta l(a, t)da. \qquad (3)$$

Expression (2) is an initial condition, and (3) is an integrated boundary condition for the equation (1). Parameter β in the equation (3) is a birth rate, which, generally speaking, depends on age *a* and time *t*, $\beta = \beta(a, t)$, and A_N is a maximal age. The main result for task (1) - (3) is the theorem of Lotka-Schwarz [1]. This theorem concludes that there is a constant "internal growth rate of a population," λ, determined by mortality rate, β, and birth rate, μ, such that if $\lambda < 0$, then $l(a, t) \rightarrow 0$ as $t \rightarrow \infty$, and if $\lambda \geq 0$, then as $t \rightarrow \infty$ normalised density $exp(- \lambda t)l(a, t)$ aspires to a "stable" marginal distribution, $l^*(a, t) = l_1(a)l_2(t)$, to such distribution, for which the age ratio does not vary in time. An example of such distribution we shall use here:

$$l(a, t) = C\, exp(\omega(t - a)). \qquad (4)$$

Indeed, the distribution (4) turns out from (1) - (3), for example, in a case when all new-born have the parents of age A_1, and the average number of children per parent is constant and is equal to α: $l(0, t) = \alpha(l(A_1, t)$, i.e. $\beta = \alpha\delta(a - A_1)$. Becoming older the age group decreases with the given rate μ: $l(a, t) = l(0, t - a)\, exp\,(- \mu a)$. The initial age distribution of the population, $l_0(a)$, obeys to a condition $l_0(a) = C\, exp(a\, ln\alpha\, /A_1)\, exp(- \mu a)$. By setting $\omega = - \mu + (ln\, \alpha)\, /A_1$, after simple calculations, we receive (4).

The rate ω is determined by balance between birth and mortality processes. In our case it defines change in size of a total population, and also age ratio, so that it is a main parameter of demographic structure that we study.

3. Investment in education and income

A part *k* of gross domestic product (GDP) of studying economic system, $Y(t)$, will be considered to spend for the investment in education, $E(t) = k(t)Y(t)$. Assume that the education of each person takes place from the time of birth, $a = 0$, up to leaving age of education, A_S. Then the number of the learners, $S(t)$, according to (4) is determined by

$$S(t) = \int_0^{A_S} l(a, t)da = C\, exp(\omega t)[1 - exp(-\omega A_S)] / \omega. \qquad (5)$$

The current expenses of education per learner at time t will be determined by

$$s(t) = E(t) / S(t). \qquad (6)$$

Assume that increase in educational level of learners at time t, $w(t)$, is completely determined by the current average expenses of education

$$w(t) = \varphi(s(t)), \qquad (7)$$

where $\varphi(s)$ is monotonically increasing function of its argument. An educational level for age group of age a at time t (i.e. for generation that was born at time $t - a$), $W(a, t)$, is defined as a total increase of this level from the time of birth:

$$W(a, t) = \int_0^{\min(a, A_S)} w(t - a + \tau)\, d\tau. \qquad (8)$$

For simplicity of calculations we assume that: 1) the labour productivity of economically active age group of age a at time t, $y(a, t)$, is directly proportional with parameter ν, the same for all age groups, to the educational level of this age group

$$y(a, t) = \nu W(a, t); \qquad (9)$$

2) there are not deficits in supply of natural resources and capacities for studying industrialised economy; 3) the emission of pollution is done while in production and its value for age group of productive age a at time t, $e(a, t)$, is inversely proportional to the educational level of this age group

$$e(a, t) = \pi / W(a, t). \qquad (10)$$

Then the GDP, $Y(t)$, is defined as

$$Y(t) = \int_{A_B}^{A_E} y(a, t)\, l(a, t)\, da = \nu \int_{A_B}^{A_E} W(a, t)\, l(a, t)\, da, \qquad (11)$$

where A_B is an age at entry, and A_E is an age at withdrawal of productive age, $0 < A_B < A_S < A_E < A_N$.

The pollution of the industrialised economy, $P(t)$, is calculated as

$$P(t) = \int_{A_B}^{A_E} e(a, t)y(a, t)\, l(a, t)\, da = \pi\nu \int_{A_B}^{A_E} l(a, t)\, da = \pi\nu L(t). \quad (12)$$

By (12), the pollution, $P(t)$, is directly proportional to the size of population of productive age, $L(t)$. According to (4) the value $L(t)$ is defined by

$$L(t) = \int_{A_B}^{A_E} l(a, t)\, da = C \exp(\omega t)\, [\exp(-\omega A_B) - \exp(-\omega A_E)] / \omega. \quad (13)$$

The size of total population, $N(t)$, according to (4) is defined by

$$N(t) = \int_{0}^{A_N} l(a, t)\, da = C \exp(\omega t)\, [1 - \exp(-\omega A_N)] / \omega. \quad (14)$$

Hence, structural parameter ω, specifying age distribution of the population (4), at the given maximal age, A_N, is rate of growth of a total population. The average income per capita is defined by

$$x(t) = (1 - k - u)\, Y(t) / N(t), \quad (15)$$

where u is a share of output Y going on productive consumption (including the investment in this sphere). The equations (5)-(15) define the age structure dependence and the education policy dependence of the well-being of the population.

4. Number of children and income

The number of children (number of persons with age $a < A_B$) at time t by (4) is defined as

$$D(t) = \int_{0}^{A_B} l(a, t)\, da = C \exp(\omega t)\, [1 - \exp(-\omega A_B)] / \omega. \quad (16)$$

Thus, total number of children (13) as well as size of population (11) is defined by parameter of demographic structure ω, which is as well growth rate for number of children. Then the average number of children on one adult, d, is completely determined by parameter ω of age distribution, $l(a, t)$, by given maximal age of children, A_B, and by given maximal age of living, A_N.

$$d(\omega) = D(t) / [N(t) - D(t)] = [1 - exp(-\omega A_B)] / [exp(-\omega A_B) - exp(-\omega A_N)]. \qquad (16)$$

The aim of our work is to define the fertility and education dependence of well-being. The average incomes x dependence of parameter of demographic structure, ω, is defined by (15) with the use of (5)-(9), (11), (14) if we make the additional assumptions about the kind of function φ of (7). After that it is possible to find dependence of the average income, x, from ratio of number of children and adults, d (see (17)), and of share of expenses of investment in education, k (see (6)).

5 Case when expenses of education are a constant share of the GDP

Assume that share k of GDP used for investment in education at time t is constant and does not depend on number of learners $S(t)$, $k = k_0$. The increase $w(t)$ of educational level of learners $S(t)$ will be considered to be equal to the average expenses of education, $w(t) = s(t)$. Then GDP can be obtained from (4)-(9), (11).

$$Y(t) = (\nu\omega k_0) / (1 - exp(-\omega A_S)) \int_{A_B}^{A_E} \int_0^{\min(a, A_S)} exp(-\omega\tau)\, Y(t - a + \tau)\, d\tau\, da. \qquad (18)$$

Expression for the average income per capita, x, can be obtained from (15), (14), and (18).

$$x(t) = (\nu\omega k_0) / (1 - exp(-\omega A_S)) \int_{A_B}^{A_E} exp(-\omega a) \int_0^{\min(a, A_S)} x(t - a + \tau)\, d\tau\, da. \qquad (19)$$

In this case average income per capita (19) varies with t and can remain constant only at the definite value of structural parameter ω determined by expression

$$(1 - exp(-\omega A_S)) / (\nu\omega k_0) = (exp(-\omega A_B) - exp(-\omega A_S)) / \omega^2 +$$

$$+ (A_B\, exp(-\omega A_B) - A_S\, exp(-\omega A_S)) / \omega. \qquad (20)$$

To examine how the average income per capita (19) depends on demographic structure (4) it is necessary to make the additional assumptions on character of change with time of this average income. A characteristic mode of change of the average income is its balanced growth, at which demographic structure (parameter ω) remains constant.

$$x(t) = x_0 \exp(\gamma t). \tag{21}$$

Parameter γ in (21) is a rate of change of the average income. For each given demographic structure such as (4) (for given parameter ω) it is possible to find the appropriate growth rate, γ, from (19), (21).

$$(1 - \exp(-\omega A_S)) / (\nu\omega k_0) = (1 / \gamma) \{[1 / \omega - \exp(- \omega A_B) / (\omega + \gamma)] \exp(- \omega A_B)$$

$$- [1 / \omega - 1 / (\omega + \gamma)] \exp(-\omega A_S)$$

$$- \exp(- (\omega + \gamma)A_E) (\exp(\gamma A_S) - 1) / (\omega + \gamma)\}. \tag{22}$$

In Fig.1 the dependence $\gamma = \gamma(\omega)$ is shown and in Fig.2 the dependence $\gamma = \gamma(d)$ is shown. The Fig.1-2 were received at the following values: $A_B = 18$, $A_S = 30$, $A_E = 60$, $A_N = 100$, $\nu = 0.5$, $u = 0.2$, $k_0 = 0.1$. The average number of children on one adult, d, (see Fig.3) and pollution, P, (see Fig.4) will grow up exponentially as the parameter ω increases. In this process the growth rate of the average income γ is reduced practically linearly (Fig. 1) as the growth rate of the population, ω, increases, and it is reduced nonlinearly as the number of children on one adult, d, increased (Fig. 2).

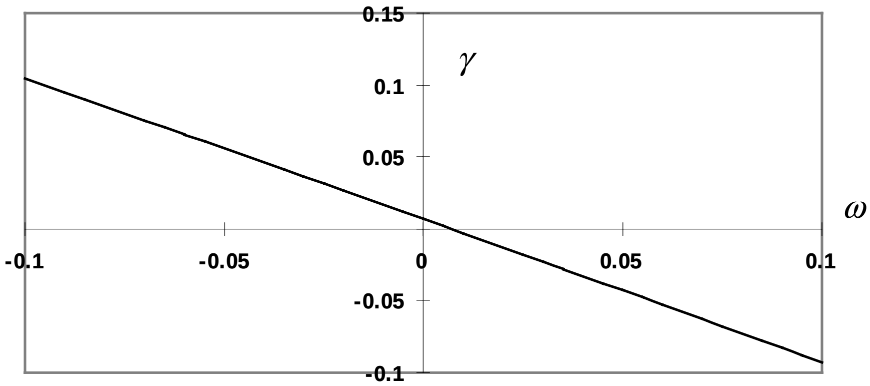

Fig.1

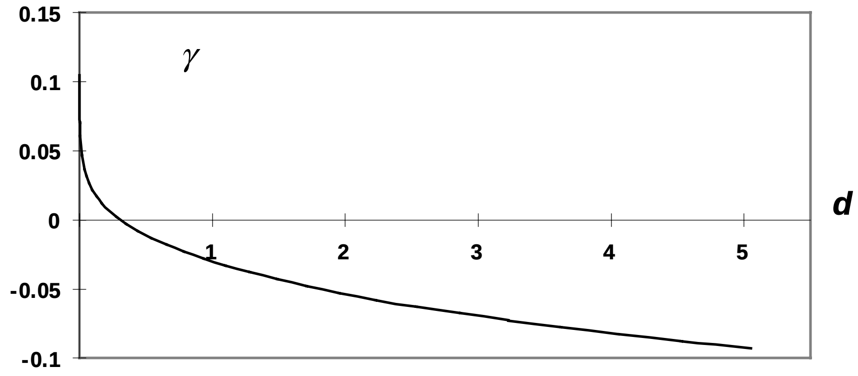

Fig.2

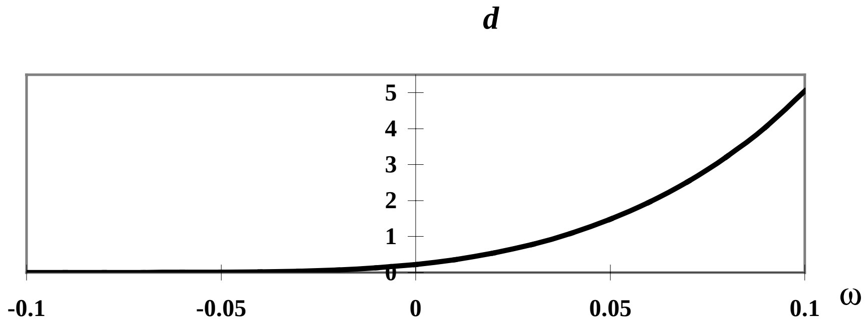

Fig.3

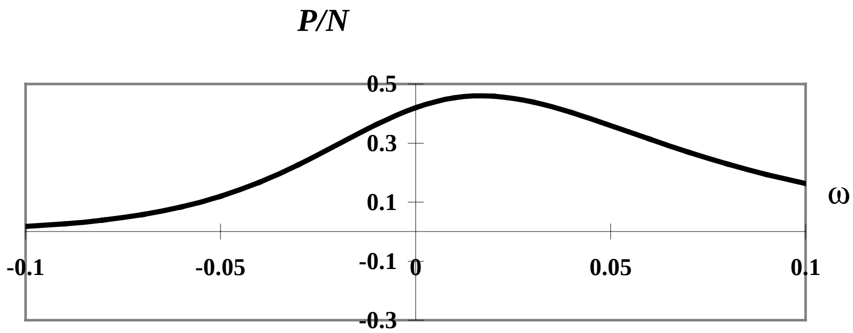

Fig.4

6. Conclusion

The given work is fragment of description of an ecological demographic economic model that is built in the Computing Centre of the Russian Academy of Sciences (see, for example, [2-5]).

The work shows that the quality of life can be improved by decrease of the population size not only ecologically but also economically.

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