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Another Tempo Distortion: Analyzing Controlled Fertility by Age-specific Marital Fertility Rate

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Another Tempo Distortion: Analyzing Controlled Fertility by Age-specific Marital Fertility Rate

Kiyosi Hirosima*

Abstract

The rise in marital fertility in East Asian societies with very low fertility has been reported through analyses using the age-specific marital fertility rate (AMFR). Though the measure is often considered related to the average number of children married women have (CMF), we demonstrated that such an interpretation is often erroneous (AMFR problem) and valid only under limited conditions in more or less controlled fertility, a fact that has been known by some researchers. We conducted numerical simulations based on a simple mathematical model. Holding completed marital fertility (CMF) constant, tempo changes in the age-specific marriage rate and in the duration-specific marital fertility produce a parallel and opposite change in the AMFR, respectively. Note that the former is in the opposite direction of demographic translation. This means that a change in the AMFR caused by such tempo changes may cancel the change in the CMF thus leading to an erroneous interpretation.

We should be careful in using the AMFR when the age at marriage or the tempo in duration-specific marital fertility changes or differs notably. Hence, the observed rise in the AMFR should be interpreted after subtracting the enormous effect by such tempo changes so as to avoid exaggeration of the marriage rate decline and negligence of marital fertility decline. This problem may even apply to some developing countries or Western societies.

1 Introduction

In the 1970s and 80s, the rise in marital fertility attracted the attention of researchers for it was seen as a contradictory phenomenon in the process of demographic transition. Today, researchers might be challenged by a similar phenomenon in societies with very low fertility in East Asia. In fact, not negligible rises in marital fertility is reported and the fertility decline there is believed to be derived exclusively from marriage postponement (a decline in the marriage rate), which confusingly leads the general public to dilute the concern over the marital fertility in each society (South Korea, Eun, 2003; Japan, Ministry of Health and Welfare, 1998; Hong Kong, Yip and Lee, 2002; Taiwan, Freedman et al. 1994). We argue that such recognition may be erroneous since it is based on analyses using a marital fertility measure, i.e. the age-specific marital fertility rate (AMFR). It may seem strange, however, to scrutinize the AMFR since it is so firmly rooted and widely used in demography and in fact, since it is indispensable for the study of societies with few births outside marriage and scarce

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data such as that of historical demography. The problem in the measure occurring in certain circumstances may not be quite unknown to some demographers (Freedman, Hermalin and Sun, 1972; Freedman and Casterline,1982, p78; Hirosima, 1986). Yet, systematic examinations have not been undertaken so that one would be puzzled about what the criteria for its appropriate application is. A thorough examination of the AMFR will not only resolve the confusion in aggregate fertility measurement in East Asian societies or potentially other Western societies with few extramarital births, but also provide insight into the earlier studies on the onset of the demographic transition in the Western societies where a similar measure to the AMFR, I_g, was widely used (see, for example, Coale and Watkins, 1986).

The major objective of this paper is to specify the cases in which the AMFR can and cannot be used for quantitative analyses of fertility. First, we review the literature and trace the almost general negligence or insufficient treatment of the problem in order to explain our approach. Then, we show that the AMFR in controlled fertility can be modeled by two functions: the marriage duration-specific marital fertility rate and the age-specific marriage rate, which means that it is determined not only by the quantum of the former but also the two tempos in both rates. Also, we show that the average number of children women have (completed marital fertility, CMF) coincides with the quantum of the duration-specific marital fertility rate for we postulate the rate as independent of marriage age in our model. We consider the CMF as the most fundamental marital fertility measure for controlled fertility. Based on this model, we examine the relationship of the AMFR with the CMF when such tempos change in order to show that the quantum of the AMFR (total marital fertility rate, TMFR) changes without a corresponding change in the CMF when either of the tempos of the two rates change. In these cases, we show that the AMFR leads to an erroneous identification of changes in the CMF, and that therefore, when the CMF is not available and we use the AMFR, it should be accompanied by the marriage rate (proportion married), not causing the deteriorating (positively correlated) change in the AMFR, given constant marriage duration-specific marital fertility rates.

Next, we examine the decomposition of the change in the period total fertility rate, TFR, using the period AMFR to know whether the AMFR shows the cause of the change when the tempos and quantums of the two rates for cohort change. In the simulation model, we incorporate the cohort-period translation or demographic translation (Ryder, 1964), assuming a change occurring to one cohort for its generality and not a continuous change as many demographers assume (Ryder, 1964; Bongaarts and Feeney, 1998; Kohler and Philipov, 2001). The demographic translation is not our main subject but elaborated enough for the simulation model because the examination of the AMFR in dynamic (cohort-period interacting) situations cannot be conducted without it. We argue that the most serious problem is the misleading change in the AMFR caused by a change in the tempo of the marriage rate without a correseponding CMF change and that therefore, we may have to avoid relying on this decomposition when we find a notable change in the mean age at marriage.

As we show, the problem in the AMFR itself, however, is derived not from the demographic translation but from its composite nature of marriage duration-specific fertility rates by different marriage age in controlled fertility. Finally, we discuss the precedent results of studies on the decline or the difference in fertility using the AMFR in some developed societies and developing societies, based upon the insights obtained by our study. Also, similar problems in the marital fertility index by Coale, I_g, to that in the AMFR will be examined in Annex C.

1.1 AMFR Problem

Period fertility has a prominent value since it forms the population of the youngest age in a year. It determines the extent of population growth and will remain a part of the age-specific population for a long time. The unchanging concern over the total fertility rate (TFR) in recent years apparently reflects this fact (for example, Bongaarts, 2002; Kohler et al., 2002).

Among period fertility indices, the period age-specific fertility rate (AFR) is obviously most important for several reasons. First, and theoretically, as far as we are interested in the age structure of population, it shows how the population age structure relates to the youngest (under one year old) population, the sum of products of age-specific population by the AFR, neglecting infant mortality. Second, and more practically, it is the simplest among basic fertility measures. Third, and because of the second reason, it is one of the only obtainable fertility indices in some modern or historical societies. Fourth, because of the second reason again, it is most understandable and influential to a wide audience in societies. The TFR that represents the AFR as its simple total shares those reasons.

The downsides of the AFR and the TFR, however, are that it is often difficult to know whether they show the trend in cohort fertility or period fertility without distortion by period fertility-related structure or by tempo change of the fertility, and which target, for example, of parity, the policy should aim at. Obviously more sophisticated fertility measures should be used for such purposes (Feeney, 1991; Rallu and Toulemon, 1994, among others).

Nevertheless, as far as we are interested in the age structure of population due to the clear meaning of the AFR and the TFR to it, we should continue to use and explain the cause of the change in them. The adjustments of TFR concerning the tempo change in fertility in recent years are one of the efforts in this line (for example, Bongaarts and Feeney, 1998; Kohler and Philipov, 2001).

Another relevant direction of the explanation concerning the change in the AFR and in the TFR is to decompose into nuptiality and marital fertility in societies where nuptiality has a direct impact on fertility i.e. societies with little births out of wedlock (for the works in this direction, see Ruzicka, 1982, for example). This can be done given data of mothers of births and all women not only by age but also by marital status, respectively. We can obtain the age-specific marital fertility rate (AMFR, $B(x)/M(x)$) and the age-specific proportion married (APM, $M(x)/P(x)$) by the data ($B(x)$, $P(x)$, and $M(x)$ denoting births by age of mother, women by age, and married women by age, respectively).

Via these two rates, if there are no births outside marriage, the AFR could be written as the multiplication of the two, AMFR and APM,

AFR= $B(x)/P(x) = \{B(x)/M(x)\}\{M(x)/P(x)\}$ =AMFR*APM

In this decomposition the first term could be interpreted as showing the marital fertility and the second, nuptiality. This decomposition could be used against fertility measures based upon the age-specific fertility rate, not only the TFR, but also the CBR, the GFR and so forth, though we are most interested in the TFR in most cases. It is because they all could be expressed using the AFR ($B(x)/P(x)$).

For this reason, the usage of the AMFR is widely found in analyses of aggregate changes or differences of those fertility indices in terms of time series, geographical or other characteristics, and in forms such as demographic decomposition, demographic standardization, simulations or regressions [1]. In this paper we deal mostly with change in time for convenience but, for generalization, we only interpret change as difference by other attributes than time (for example, geographic difference or difference by educational attainment).

In such analyses, the crucial problem is whether the AMFR is really relevant for marital fertility measurement. This seems to be a serious problem that needs fuller examination than has yet been done. We refer to this as AMFR problem and examine ways dealing with it in this paper.

1.2 How has AMFR been described?

Reviewing comments on the AMFR in demography, we find optimism at the introduction by the founders of the standardization of crude birth rate using AMFR. Newsholme and Stevenson (1905) wrote, "[the standardization] correct[s] completely both for the varying proportion of married women in compared population and for the varying fertility at different periods of married life". Barclay (1958) apparently followed this optimism, saying, "Sometimes age-specific rates are computed separately for married women, as a further refinement when the effect of variations in marriage patterns must also be excluded (p.172)". This attitude seems to have been basically inherited by standard demography textbooks, such as Shryock, Siegel et al. (1973), the United Nations (1989) and Preston et al. (2001). Shryock, Siegel et al. (1973, p.486) only criticizes summing up the AMFR in order to make the total marital fertility rate (TMFR) because of its same weight on youngest ages in the TMFR. It introduces instead the standardization of the TFR for marital status and the AMFR. The textbook by Preston et al. (2001) only warns of the conceptions outside marriage concerning the AMFR.

The most famous work that uses the AMFR may be the model fertility by Coale and Trussell (1974) as they produce model age-specific fertility rates on the base of the AMFR. Their work was founded on the work of Henry (1971, 1976) who deduced that in natural fertility, the AMFR is independent of the age at marriage. This observation was actually an approximation but still well-verified [2]. When comparing modern controlled fertility with natural fertility, he used data only of a marriage cohort of British "women married at about 20 years old around 1920" and not a birth cohort or a synthetic cohort (period) of different marriage ages. Coale and Trussell (1974) extended the usage of the AMFR to the period AMFR that derived from women of different marriage ages, neglecting the effect of marriage age distribution on the AMFR in modern fertility with various degree of control. They were only concerned about reproducing age-specific fertility rates of all societies and comparing the extent of fertility control among populations before the end of a demographic transition and they had no concern as to whether the AMFR represents the over-all marital fertility level or not.

In contrast to the general negligence of the AMFR problem in demography, there have been a small number of demographers who are critical about using the AMFR for deliberately controlled fertility (Tachi, 1936 [3] ; Pressat, 1972). "In computing the age-specific marital fertility rates [AMFR] regardless of age at marriage, the relationship between fertility and duration of marriage is neglected, and the result, in spite of an apparent refinement, is a quite broad description of fertility. " (Pressat, 1972, p. 181) And according to Pressat (1972), the only usefulness of the AMFR is to distinguish whether the fertility is deliberately controlled or not by its age pattern (p.185). We think, however, that the AMFR can be used for quantitative measurement in certain situations for controlled fertility as well, and specifying such situations is the main objective of this article.

Marriage age influence on the AMFR was observed by the paradox of higher AMFRs for more educated women than for those less educated because of a shift in marriage age (Sutton and Wunderlich, 1967; Bogue, 1969, p. 711) though they also used the AMFR as a marital fertility measure without taking into account marriage age itself. The problem is more noticeable than before in the works on decomposing changes in the TFR or the CBR (Feeney et al., 1989; Zeng et al., 1991). They correctly pointed out the effect of marriage age on the AMFR but seemingly characterized it as owing to cohort-period translation citing Ryder (1980) which did not discuss the AMFR, and commented that decreasing age at marriage gives upward pressure to the AMFR (Zeng et al., 1991). Of course, we do not deny the increase in the TFR or the CBR caused by a decrease in age at marriage through demographic translation (Ryder, 1964) (For an interesting question of demographic translation for the CBR suggested by Feeney et al. (1989) and Zeng et al. (1991), see Appendix B). We will argue that the decrease in marriage age potentially causes the *decrease* in the AMFR at the same time.

Increases in the AMFR in some developed and developing societies undergoing the fertility decline with few extramarital births have been reported (Eun, 2003; Yip and Lee, 2002; Chang, 2003; Freedman et al., 1994; Freedman, Hermalin and Sun, 1972; Cho and Retherford, 1973; Coale, Goldman and Cho, 1982; Retherford and Rele, 1989). Apparently, these may be caused by an increase in marriage age. Besides, advancements in tempo of duration-specific marital fertility in developing societies have been substantiated (Donaldson and Nichols, 1978; Freedman and Casterline, 1982; Rindfuss and Morgan, 1983). Such changes in the AMFR by the change in tempo of duration-specific marital fertility seem to have not yet been explained by fertility models. We will attempt to do this here.

2 Relationship between AMFR and CMF in two fertility regimes

In this section, let us consider fertility for a cohort for its straightforwardness of theoretical reasoning. (If all vital rates are constant in time, the whole discussion holds for the period study too [4].) The most representative measure of marital fertility is the average number of children of married women, which we call completed marital fertility (CMF), meaning completed fertility for married women. Age-specific marital fertility (AMFR) is often considered a measure related to this. The problem is what relationship the AMFR really has to the CMF. As we think about this theoretically, we make mathematical models for controlled fertility and natural fertility.

An example of age-specific marital fertility rates (AMFR) in a natural fertility regime is shown as five convex curves with different marriage ages from English parish data in Figure 1 (Wilson et al., 1988). Evidently we may regard the AMFR at every age is as the same, notwithstanding marriage age in natural fertility, since the difference between the curves is comparatively small, though they are apparently higher at each age for those married at older ages. This attitude is the same as most historical demographers have had.

As an example of the AMFR in a controlled fertility regime, that of women married at 20-22.5, 25-27.5 and 30-32.5 years old in Great Britain in 1919 (Henry, 1976) is also depicted in Figure 1. The total of these rates (multiplied by 5) makes the CMF. The age pattern shows the typical concave curve concentrated to the first ten years of marriage duration. Note that each AMFR is at the same time a duration-specific marital fertility rate, with marriage age being the same for the cohort.

We can interpret the pattern of fertility in this regime that it is deliberately controlled to achieve a normally desired number of children in a certain short period so that it is strongly decided by marriage duration rather than by age itself of married women, as has been well-known (Page, 1977; Takahashi, 1979; Kono et al., 1983). We can see larger relative differences in the AMFR over the ages by marriage age for controlled fertility than that for natural fertility. Based upon these characteristics, we could model that the marital fertility rate has a similar duration-specific pattern irrespective of marriage age in a controlled fertility regime.

In addition to these marital fertility rates by marriage age, we could place the AMFR with all marriage ages aggregated. This is the period AMFRs of Japan in 2000 (broken line with small circles) instead of the cohort AMFR, for convenience. Similar age-patterns in different countries are shown in, for example, Shryock, Siegel et al. (1973, p.478). From this figure, it may be easy to understand that the aggregate AMFR at each age interval is a weighted sum of these duration-specific fertility rates by marriage age and that this AMFR can be obviously affected by the composition of marriage ages in each age interval. The aggregate AMFR is the subject of this paper.

2.1 Deliberately controlled fertility

For the mathematical expression of deliberately controlled fertility, we simply set *g(t)* as the marriage duration-specific marital fertility rate where *t* or *x-a* is marriage duration, *x* being the women's age and *a* being age at marriage [5]. The expression of marital fertility, *g(t)* means that marital fertility is only decided by *t* (=*x-a*), duration of marriage and not by *a* or *x*. Actually, the marital fertility by marriage duration is known to become slightly lower over the duration of marriage as age at marriage gets older (see Figure 1; Henry, 1976; Inaba, 1995; Billari et al. 2000). We exclude this effect and set marital fertility independent of marriage age. This simplification enables us to understand more clearly the effects of marriage age and duration-specific marital fertility on the AMFR, separately. We will give notice to this simplification later as necessary (Section 3.1). Such effect as of marital fertility (*g*(*t*)) decline by delayed marriage can be modeled on the combination of the effects by the two causes in our model (For decline of marital fertility, see at the end of Section 2.1 and Simulation B1, and for delayed marriage, see Simulation A4-A5 and B4).

We also set *m(x)* as the age-specific marriage rate, and *P* as the size of the female cohort. *P* does not change by age because we neglect mortality. Then *M(x)*, the number of ever-married (= currently married) women at age *x* is expressed by the integration of *m(a)*, as formula (1).

$$M(x)=P\int_0^x m(a)da \qquad \ldots\ldots(1)$$

where we assume that no divorce and no widowhood occurs so that every marriage is a first marriage. *B(x)* or births to women of age *x* is expressed by the integration of births of women married at the age from 0 to *x*, that is an integration of *g(x-a)*, weighted by the marriage density *m(a)*, as formula (2).

$$B(x)=P\int_0^x m(a)g(x-a)da \qquad \ldots\ldots(2)$$

From these two, by dividing (2) by (1), we get the AMFR.

$$\text{AMFR}=B(x)/M(x)=\frac{\int_0^x m(a)g(x-a)da}{\int_0^x m(a)da} \quad \text{…………………………………………………(3)}$$

This formula shows that the AMFR is a weighted average of *g(x-a)*, weighted by *m(a),* from age 0 to *x* as we explained for Figure 1 [6].

The TMFR is the total of the AMFR over all ages. This is represented by formula (4).

$$\text{TMFR}=\int_0^{\omega}[B(x)/M(x)]dx$$

$$=\int_0^{\omega}\frac{\int_0^x m(a)g(x-a)da}{\int_0^x m(a)da}dx \quad \text{………………………………………………… …………(4)}$$

Meanwhile, the CMF, the completed marital fertility rate is a ratio of the total births of a cohort of women to the number of married women of the cohort at the end of reproductive age, ω, as we postulated that there are no deaths and divorces.

$$\text{CMF}=\int_0^{\omega}B(x)dx/M(\omega) \quad \text{…………………………………………………………...(5)}$$

Note that in spite of the similar appearance, the TMFR and the CMF are completely different as shown in (4) and (5). Substituting *B(x)* and *M(x)* according to formulas (1) and (2), CMF is expressed as follows and then we exchange the order of integration. The range of integration can be treated as from 0 to infinite. Then we can divide the integration into a multiplication of the two parts and cancel the integral of *m(a)*, eventually leading to the expression (6).

$$\text{CMF}=\frac{\int_0^{\omega}\int_0^x m(a)g(x-a)dadx}{\int_0^{\omega}m(a)da}=\frac{\int_0^{\omega}\int_a^{\omega}m(a)g(x-a)dxda}{\int_0^{\omega}m(a)da}=\frac{\int_0^{\omega}m(a)\int_0^{\omega}g(t)dtda}{\int_0^{\omega}m(a)da}$$

$$=\int_0^{\omega}g(t)dt \quad \text{……………………………………………….…..……..(6)}$$

Thus we can say that the CMF equals the integration of *g(t),* i.e., the total of marriage-duration-specific marital fertility rates [7]. In other words, the quantum of *g*(*t*) is equal to the CMF in our model.

In conclusion, according to formulas (3), (4) and (6), if the marriage rate, *m(x)* is fixed and marital fertility becomes $kg_0(t)$ from $g_0(t)$, meaning with no change in the duration-specific pattern of marital fertility but only the level is changed by *k* times, then the CMF will increase by *k* times and the AMFRs and the TMFR will also increase by *k* (constant). This change in *g(t)* described above can be defined here as "quantum change without tempo change". The term will be used hereafter in the meaning defined in this way.

In the same manner, given *g*(*t*) fixed, if *m*(*x*) changes in quantum and not in tempo, then the AMFR (the TMFR) and CMF do not change at all according to the same formulas.

Hence, we can conclude more practically that the AMFR shows the change of the CMF only in the special case where the tempos in *m(x)* and *g(t)* are both fixed. Conversely, the change in tempo without change in quantum of *m*(*x*) or *g*(*t*) may cause changes in the AMFR and in the TMFR as shown in formulas (3) and (4) but not in the CMF. In this case, the AMFR cannot be used to show the level of the CMF. Such relations will be examined through numerical simulations in the next section.

2.2 Fertility with little voluntary control

Concerning natural fertility or fertility with little voluntary birth control, we express the marital fertility rate as $G(x)$, instead of as $g(x\text{-}a)$, which is neither decided by age at marriage, a, nor by marriage duration, t or $x\text{-}a$. This is the assumption we made, based on the finding by historical demographers (Figure 1), and also adopted by others [8] (Page, 1977; Trussell, 1979, 1981; Trussell et al. 1982). The AMFR is, by definition, $B(x)$ divided by $M(x)$. Substituting these by formulas (1) and (2) in the same way as controlled fertility, this reduced to $G(x)$.

$$\text{AMFR}= B(x) \,/\, M(x) = \frac{\int_0^x m(a)G(x)da}{\int_0^x m(a)da} = \frac{G(x)\int_0^x m(a)da}{\int_0^x m(a)da} = G(x) \quad \ldots\ldots\ldots\ldots\ldots\ldots..\ldots\ldots(7)$$

Though it is so obvious by the definition of $G(x)$ itself, we followed the same definition made for controlled fertility.

Using this we get the TMFRa, the average number of children in the lifetime for women who get married at age a. It is the total of age-specific marital fertility rates from age a to the end of the reproductive period. This minimum age, a, is usually set as 20 years old for comparison.

$$\text{TMFR}a = \int_a^\infty G(x)dx \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..\ldots(8)$$

On the other hand, CMF (completed marital fertility) is expressed by formula (9).

$$\begin{aligned}\text{CMF} \\ &= \int_0^\omega B(x)dx \,/\, M(\omega) \\ &= \frac{\int_0^\omega \int_0^x m(a)G(x)dadx}{\int_0^\omega m(a)da} = \frac{\int_0^\omega G(x)\int_0^x m(a)dadx}{\int_0^\omega m(a)da} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(9)\end{aligned}$$

In fact, using this measure, CMF is very inconvenient to represent the level of marital fertility because it depends on marriage distribution $m(x)$, and $m(x)$ can not be separated from CMF though we usually want to show the marital fertility independent of marriage distribution. That may be the reason why historical demographers use the TMFRa in (8) rather than the CMF as representative of marital fertility in a society with little voluntary fertility regulation in historical literature. This practice seems to have affected the studies of controlled fertility as well, leading to careless usage of the AMFR or the TMFR, instead of the CMF in (6) for fertility with variant control. Nonetheless, the AMFR shows the level of the marital fertility as a part of the TMFRa if we assume the TMFRa is a representative measure of marital fertility in a population with little deliberate fertility control.

We have modeled the two fertility regimes as strictly different in mathematical expression. We cannot, however, utterly deny that natural fertility also has a duration specific characteristic to some extent, as shown in Figure 1. In fact, if the duration-specific marital fertility, $g(t)$, keeps high values for a fairly long marriage duration, meaning with weak duration-dependent fertility control, then the AMFR approaches to almost the same for different marriage ages, which is similar to natural fertility since the range of marriage age is limited to certain ages relatively shorter than reproductive ages [9]. Hence natural fertility also could be practically expressed by the same mathematical expression if $g(t)$ could be numerically set to reflect the reality. On the contrary, the sheer problem may arise when marital fertility is expressed by the AMFR for controlled fertility. In practice, we may face the transitionary phase of the two fertility regimes in particular societies and encounter a problem whether or not the AMFR can be used for marital fertility measure. The key for the problem would be the comparative observation of the AMFR with the CMF and $m(x)$ instead of using the AMFR alone if $g(t)$ is not obtainable.

3 SIMULATIONS

3.1 Simulations of AMFR: How does AMFR relate to CMF?

Hereafter, we mainly assume modern controlled fertility though the simulations here could be also valid to a certain extent for fertility with little control if its marital fertility has characteristics to some extent duration-dependent as mentioned in the last part of the previous section (for a simulation of natural fertility as modeled in the last section, see Appendix A).

In this section we conduct static simulations of the AMFR to find out the relationship between the AMFR and the CMF (or quantum of $g(t)$) basically both for a cohort, which we call "static" because we deal with only cohort rates (or period rates provided all vital rates are constant in time) rather than both period and cohort. We have already demonstrated in the last section that change in the AMFR or in the TMFR can show the change in the CMF only if tempos in $m(x)$ and $g(t)$ are fixed through the expression in the formulas (3) , (4) and (6). We will specify the relationship by showing how the AMFR changes (using formula (3)) when either of $m(x)$ or $g(t)$ is fixed and the other differs in tempo but not in quantum. Tempo is defined here simply as the mean age of the function, as the higher moments of the two functions can hardly influence the AMFR.

We conduct five simulations; first, A1: base simulation, A2-A3: the postponement of marital fertility, and A4-A5: marriage postponement. Note that forward shift or advancement is the reverse of postponement, for which we need only to interpret the result reversely without additional simulations. The result of quantum increase in $g(t)$ or $m(x)$ without change in tempo is already explained in Section 2.1 and need not to be simulated.

First, we set input data for the base simulation. The input is the hypothetical data shown in Table 1 which describes the process of marriage and childbearing for a cohort that is 15 years old in 1985 [10]. The cohort experiences these events as it grows old in the following years. As all the variables should be discrete in a simulation model, vital rates, $m(x)$ and $B(x)$, are measured in the age interval x and the marital status, $M(x)$, is measured at the beginning of the age interval of x. We set the data to represent the process as compressed as possible in time to make it concise, but also so as not to lose the essential features of reality. The single year in age and in calendar year is adopted to show effectively the mechanism of demographic translation in dynamic simulation in the next section. Accordingly, age schedule is compressed to 11 years (15-25 years old) [11] rather than 35 years (15-50 years old) to avoid needless tediousness, though the level of rates remain realistic.

The column $m(x)$ shows the age-specific marriage rates. The age-specific births, $B(x)$, represents here fertility rate, since we set the number of women, N, as one for convenience (interpret it as a unit of a large number). It is generated by the formula (2) [12], using the marriage duration specific fertility rate, *g(t)*, which is assumed to be (1,1,0), meaning that at the duration of first year, the fertility rate is one and at the duration of the second year, one, and the third year, 0 (Table 2). We can interpret this that all women in the cohort have their first birth one year after the marriage and their second birth two years after the marriage. This high value of marital fertility, *g(t)*, is to adapt to compressed ages and to represent the concentration at shorter duration and mostly monotonous decrease, as shown in Figure 1. (See concentration of birth rate in duration of marriage of 5 years, for example, for Australian women married in 1911-65 in Table 5.5 in Newell, 1988.) Meanwhile *M(x)*, the proportion married is directly derived from accumulating the marriage rate, *m(x)*, from age 15 to each age *x*.

By dividing the fertility rate, *B(x)*, by proportion married, *M(x)*, we get the AMFR at each age *x*. The AMFR at age 17 and afterward is 1.0, 1.0, 0.83, 0.7,…, monotonously declining. It shows a maximum value of 1.0 at age 17 because at this age the births are all from women married at age 16, all of whom have had their first child. At age 18, similarly, it becomes 1.0. After those ages, those married with longer marriage duration will increase, making the AMFR decrease continuously. The TMFR summing the AMFR over ages is calculated as 4.98. These four variables by age are shown in Figure 2 (Simulation A1), too.

Note that the proportion married, *M(x)*, should always be monotonously increasing irrespective of the shape of *m(x)* and that the AMFR also, unless the shape of *g(t)* has a very low value immediately after marriage (for example, *g*(*t*)=(0,1,1), Simulation A3, Figure 3) which is not likely in reality.

The tempo of *g(t)* is set to be slightly postponed in Simulation A2 (*g(t)* =(1, 0, 1)) and further postponed in Simulation A3 (*g(t)*=(0,1, 1)) instead of (1,1,0). Thus generated, the AMFR at most ages except older ages drops dramatically, as shown in Figure 3, resulting in an overall decline and the TMFR becomes 4.51 and 3.12, respectively, which is smaller than that in the base simulation (4.98). In the same manner, if the *g(t)* is shifted backward as childbearing starts after age γ (24 years old in our simulations) or the end of marriage occurrence age, then the TMFR will be 2, equal to CMF because the denominator of the AMFR, *M(x)*, is all unity after age γ. On the contrary, if marital fertility is the earliest as *g(t)* =(2,0,0), the TMFR becomes 6.37 which may be the maximum for CMF=2 (the graph not shown). Thus, the earlier the tempo of marital fertility (*g*(*t*)), the greater the AMFR at most ages except older ages. These are the results caused by the fact that the denominator of the AMFR, the proportion married is always a monotonously increasing function. Hence, the AMFR differs enormously depending on the relative position of births (decided by tempo of *g(t)*) with *M(x)* even if the quantum of *g(t)* is constant.

Next, we postpone the marriage rate *m*(*x*) so that the mean ages at marriage, MAM becomes 20.4 years in Simulation A4 and 21.0 years (one year delayed) in Simulation A5 from 20.0 years in the base simulation (A1, Table 1). The results in Figure 4 show that, though the AMFR does not decrease at youngest ages, it increases at most ages, foming the similarly monotonously decreasing shape, resulting in augmenting the TMFR to 5.37 (A4) or 5.98 (A5) from 4.98. Note that the increase at oldest ages will be smaller if we take in the decreasing effect in *g*(*t*) by increasing age at marriage. Nonetheless, the overall increase in AMFR cannot be denied. These increases in the AMFR at most ages except the youngest ages are caused not merely by the smaller denominator, the proportion married than the base simulation at the same ages. They are also caused by the fact that there are more women of shorter marriage duration than the base simulation at the same age, yielding a higher birth rate resulting in more births. For example, the percentages of one-year marriage duration are 50% (A1), 57% (A4), and 67% (A5) at 19 years old. This positive relationship of the AMFR at most ages with age at marriage is stated as the marriage duration effect and corroborated by some authors (Freedman and Casterline,1982, p78; Hirosima, 1986).

Simulation A4-A5 also implies that, comparing with baseline Simulation A1 reversely, if $m(x)$ is shifted forward in tempo, the AMFR decreases without any change in $g(t)$. The earlier the tempo of marriage rate, the smaller the AMFR at most ages. This fact shows that $m(x)$ or $M(x)$ (proportion married) and the AMFR are not independent of each other. This may be obvious if we look into the formula (3). As the change in $m(x)$ is so common when we analyze fertility changes, the requisite of independence of the two for the analysis often mentioned in the textbook (e.g. United Nations, 1989) can be rarely satisfied in more or less controlled fertility.

In sum, what the results imply in the observation by the AMFR of fertility decline (at most ages except older ages) in the second and possibly even in the first demographic transition is as follows. If we find an increase in the AMFRs, if it is accompanied by an increase in the average age at marriage (in other words, the changes in the AMFR and age at marriage are in the same direction), as Simulation A4-A5 describes, we should doubt that it may be caused by an increase in age at marriage without an increase in quantum of marital fertility ($g(t)$). In contrast, if the increase in the AMFR is accompanied by a constant or decreasing average age at marriage (in other words, the changes are the opposite of each other), we can say the AMFR shows the real quantum increase in marital fertility, $g(t)$, or only its forward shift shown in Simulation A2-A3.

Conversely, if we find a decrease in the AMFR and it is accompanied by a decrease in the average age at marriage (changes in the same direction), then it is doubtful that a decrease in the AMFR does not show the real decline in marital fertility ($g(t)$) but that it is only caused by a decrease in marriage age, as in Simulation A4-A5. In contrast, if it is accompanied by a constant average age at marriage or even an increase in it, the AMFR shows the real decrease in marital fertility, $g(t)$, or only its postponement, similarly.

3.2 Simulations of decomposition: Can AMFR identify the cause of fertility change?

3.2.1 Period rates from cohort rates

We conduct dynamic simulations to show what relationship the AMFR, over a period, has with the quantum of marital fertility, the CMF or integral of $g(t)$, for cohorts in the analyses of change or difference in period fertility such as decomposition of change in the TFR (by formula in note 1) (for CBR and I_g, see Appendix B and C). How does the period AMFR express marital fertility when cohort $m(x)$ or $g(t)$ changes over time, when it is used in such analyses? As we manipulate $m(x)$ or $g(t)$ in simulations, we can see whether the analyses using AMFR correctly identify the causes of the changes or not. Note that though we use a continuing change in time in simulations for the explanation of dynamic nature, only two rates on different points are needed for decomposition.

We make the model in the direction from cohort to period because it is easier to describe analytically and to understand, irrespective of the real causal relationship between the two dimensions. Based upon the formulas for a cohort in the previous section, we will model dynamic processes in period dimension. It is a numerical simulation model expressed by spread sheet software, based on a rather simplified structure but with both cohort and period dimensions which some authors expressed with more specified mathematical formulas (Inaba, 1995). Our model produces the period results with a set of cohort input so that we can observe the mechanisms of demographic translation (Ryder, 1964) which we will explain. This dynamic structure of the model is based on the structure of a micro-simulation model for fertility (Kono et al., 1983). The calculations by Trussell, Menken and Coale (1982) (Figure 5, for example), based on cohorts with different parameters, are for the same purpose but without an explicit dynamic mechanism of cohort-period translation.

In the dynamic simulations the same vital rates shown in Table 1 first continue for cohorts and suddenly change from a cohort. The changes are in either the quantum or the tempo of the duration-specific marital fertility rate ($g(t)$) or the age-specific marriage rate ($m(x)$) of cohorts, making four cases: (B1) a decline in the marital fertility rate, (B2) a postponement of the marital fertility rate, (B3) a decline in the marriage rate, and (B4) a postponement of the marriage rate (Figure 5 through Figure 8).

These cases all cause eventual or temporary fertility decline. Other cases that cause fertility rise (marital fertility rise, forward shift of marital fertility, marriage rate rise, and marriage rate forward shift) are the reverse of these cases so that we need not do additional simulations. In this sense, any change in cohort fertility rates can be expressed by the combination of these four cases.

Table 3 and Table 4 show the mechanism of the model converting cohort rates to period rates. We take the example of Simulation B4, because of its most interesting nature (tables for other simulations are not shown but the mechanism is the same). In the simulations we start from cohort rates, first in Table 3, the age-specific marriage rate $m(x)$, and calculate the age-specific fertility rate $B(x)$ from marriage-duration specific marital fertility $g(t)$=(1,1,0) as in static simulations. Meanwhile, in Table 4, $M(x)$, the APM (age-specific proportion married), and the AMFR are calculated by the same procedure as in static simulations.

In Table 3, the TFR is calculated by summing $B(x)$ for each year. The decompositions of change in the TFR in each year from 1984 (one year before the beginning of the cohort change) are conducted according to the formula shown in note 1. The change in the TFR and the contributions by the changes in the AMFR (all age summed) and APM (proportion married) (all age summed) are shown in Graph (A) and Graph (B) of Figure 5 to 8 for four simulations.

In each table, rates for a cohort are represented by a diagonal series of figures and the period measures are presented on the vertical series. These tables are schematic Lexis diagrams that show the relationship between cohort rates and period rates (Ryder, 1964; Morgan, 1996). The diagram is an approximation since the time-age square does not exactly correspond to the cohort-age parallelogram and this is exactly the case when the unit is infinitesimal. Apparently, when all the rates are constant in time, these two series are identical to each other.

We assume that the change begins with the cohort aged 15 in 1985, which is shaded in the tables. In the case shown in Table 3, this cohort suddenly shows the change in the marriage rate, postponed by one year. And the cohorts succeeding this cohort followed the same schedule. Other changes in other simulations occur in the same way as this. As the real changes occur more gradually, this model may seem to be unrealistic but can be easily expressed and is ideal in showing the mechanism of changes.

Before we get in the result of decomposition by simulations, we present a formalization of a demographic translation by our model.

3.2.2 Tempo change in cohort

As we observe period rates when cohort vital rates change, the change in cohort reflects period rates in a certain way, and vice versa, which is called the demographic translation (Ryder 1964). In the simulations, we set the tempo of fertility for cohort to change in two cases, B2 and B4, where tempo in the marital fertility rate, $g(t)$, or tempo in marriage rate, $m(x)$, for cohort delays, respectively. In these simulations, the quantum of these rates for the period temporarily decreases by demographic translation. In fact, in Table 3, the total marriage rate (TMR) decreases from 1 in 1985 to 0.8 in 1989-90 and again recovers to 1 in 1994 although the cohort ever-married rate (quantum of $m(x)$) always remains 1 and only its tempo is set to be postponed from the cohort of 15 years old in 1985. This effect in marriage rate is eventually reflected in the AFR and the TFR.

Note that this cohort change (in tempo in this case) at one time can be the most fundamental by our model (also, used by Morgan, 1996) though most authors have been interested in continuing change, which fits to calculus (Ryder, 1964; Bongaarts and Feeney, 1998; Kohler and Philipov, 2001). Also, note that the decrease in the TMR in each year caused by a one-year delay among cohorts is identical to each value of $m(x)$, from youngest age to oldest, i.e. 0.05, 0.10, 0.15, 0.20, … , and the maximum decrease is the maximum value of $m(x)$, 0.2 in 1989-90 in this case. Hence, the total loss in TMRs during the temporal decline (1986-1993) is obviously equal to $\Sigma m(x)$, ($M(\omega)$) or the original TMR of cohort. In this simulation it is set as 1.0.

This is the result caused by a one-year postponement. In general, the d year postponement in the marriage rate will cause the total decrease of $dM(\omega)$, meaning d multiplied by $M(\omega)$, over the period of temporal decline, where d can be positive or negative and the integer or decimal (for the derivation of the relation, see Appendix D). We argue that this can be the most fundamental and practical relation: what amount of quantum change in total in the period quantum measure will be caused by any change in tempo of cohort rates [13].

3.2.3 Simulation of decomposition

We simulate decomposition of change in period TFR (by formula in note 1) as an example of analyses using the period AMFR because its essence of the analysis is the same with other analyses such as standardization, simulation etc. since the method of measuring the contribution of a factor is basically by the difference between the real result and the hypothetical result composed of the hypothetically differentiated value of the factor.

In Simulation B1, shown in Figure 5, the duration-specific marital fertility, $g(t)$, suddenly changes to (1, 0.6, 0) from (1,1,0) from the cohort 15 years old in 1985. No change in $m(x)$ is supposed to occur. In fact the sudden decline in cohort marital fertility from 2.0 to 1.6 causes the gradual decline in the period TFR from 2.0 to 1.6, shown in the lower part of graph (A).

The upper part of graph (A) shows the changes in the mean age at marriage (MAM) and the mean age at birth (MAB). There is no change in MAM because there is no change in $m(x)$ but there is a slight rise and decline in MAB. The eventual decline is easily understood as the relative increase in the first births due to the decrease in second births. The temporal rise may seem to be strange but is actually caused when earlier cohorts (older than the cohort 15 years old in 1985) bear more second births than younger cohorts. Thus it should be noted that a change in quantum of cohort rates causes a temporary opposite change in tempo in the period rates (see Appendix E for more detail).

Graph (B) shows contributions to the decline in the TFR from 2 to 1.6, -0.4. Here it is totally attributed to the decrease in marital fertility expressed by the period AMFR, signified by the line completely overlapping that of the TFR at any point in time. We may assess the decomposition to be successful by itself (if we do not compare it with Simulation B2). This may not be self-evident from the formulas (3) and (6) in Section 2.1.

Simulation B2 shows the result of postponement of cohort marital fertility $g(t)$ which changes from (1,1,0) to (0, 1, 1), its quantum being constantly 2. In fact, Figure 6 shows a gradual rise in the mean age at birth by one year, from 21.5 to 22.5, due to the one year delay in marital fertility. The TFR shows a temporary reduction from 2.0 to minimum 1.6 due to cohort-period translation through postponed $B(x)$ for cohort. Note that this maximum decline by 0.4 is equal to maximum of AFR, 0.4 (see Table 1), and that the total decrease in the TFR from 1986 to 1996 is 2.0 as suggested by the formula (in Section 3.2.2) as the cohort total fertility rate (completed fertility, CF) multiplied by 1 (year).

This cohort-period translation effect is expressed in the decomposition by the negative contribution of the change in the AMFR to that in the TFR. Fortunately, the superficial decrease in the AMFR without change in the quantum of $g(t)$ can play the role of depicting the decrease in the TFR by the demographic translation even if it cannot show the real cause of the change. Note that the TMFR by period shows a temporary decrease by the translation and an eventual decrease by postponed $g(t)$ without a quantum change (table not shown but suggested by Figure 3). The decomposition shows that the TFR decline is attributed totally to the change in marital fertility expressed by the AMFR at any point in time. In this sense, we may assess this decomposition to be successful by itself. Given the whole process of temporal change in the TFR, we will understand that the cause of change in marital fertility $g(t)$ of cohort is only in tempo not in quantum.

However, when we are only given the period rates and the decomposition, can we distinguish B1 and B2? Especially for B2, if we look before 1992, when the TFR is the minimal? We may argue that decomposition by period rates may not be decisive as to whether the decline in the TFR is caused by quantum (B1) or by tempo (B2) in marital fertility. If, however, we carefully look into the change in mean age at birth (MAB), we may realize the change by the former is very small.

Simulation B3 shows the result of a decreased cohort marriage rate, $m(x)$, where the quantum of marriage rate, or proportion ever-married $M(\omega)$ (=$\Sigma m(x)$) declines suddenly from 1 to 0.8. No change in marital fertility, $g(t)$, occurs. Figure 7 shows that this change causes the gradual decline in TFR from 2 to 1.6 and the decomposition attributes the decline totally to the marriage decline. We can assess this as successful by itself. Note that mean age at marriage (MAM) temporarily rises by quantum decrease without any change in tempo in

$m(x)$, which is the same phenomenon as explained for the marital fertility rate in Simulation B1.

Simulation B4 simulates the postponement of cohort marriage rates. No change in marital fertility, $g(t)$, occurs again. We give one year delayed values of $m(x)$, making the MAM increase eventually from 20.0 to 21.0. Figure 8 shows the results which we have already seen in Table 3 and 4. The MAB also increases from 21.5 to 22.5 by one year. The TFR temporarily declines from 2.0 to minimum 1.6 according to the postponement of births for cohort caused by postponement of marriages for cohort. This is a cohort-period translation effect. Again, this maximum decline, 0.4, equals the maximum value of the AFR. Note that no cohort-period translation occurs in the AMFR as shown in Table 4 because the AMFR maintains high values at youngest ages and does not shift to higher ages (see Figure 4, too). Graph (B) shows that the AMFR contributed positively to the TFR change and the APM (proportion married) contributed negatively to the TFR change, though the quantum of cohort marital fertility ($g(t)$) remains constant. This result may be expected by static Simulation A4-A5 (Figure 4) in the previous section. Also, if we look into Table 4, we understand that the AMFR actually increases mostly at each age after the marriage postponement starts. But of course, we cannot take it as an overall increase in marital fertility since $g(t)$ is fixed. Thus we should say this result of decomposition is a failure.

However, if we are given only period data before 1991 when the TFR is minimal, the result is very similar to the case of the decline in the marriage rate (B3). We argue that decomposition by period rates may not be able to decide whether the decline in the TFR is caused by quantum (B3) or by tempo (B4) in the marriage rate without knowing the marriage age change.

Note that the direction of changes in age at marriage (+) and the TFR (-) in demographic transition is opposite but the direction of changes in age at marriage (+) and the AMFR (+) is in the same direction as shown in Simulation B4.

Since we may face in reality the mixture of the causes described in the four simulations, it is important whether we can identify the cause by the decomposition. If we compare the results of decomposition shown in Graph (B) of each simulation, the cause of the change in the TFR is almost identifiable as the change in marital fertility (B1, B2) or marriage (B3, B4), if we are given the result in B4 a little before 1991 when the TFR is minimum. As we mention above, determining whether it is tempo or quantum, or to what extent, within marital fertility and marriage, is impossible without the observation on the MAM and the MAB. The most serious problem in the decomposition is the misleading rise in AMFR caused by the change in tempo in the marriage rate ($m(x)$) as shown in Simulation B4. For example, when declining marital fertility (B1) and postponed marriage (B4) are combined, the decline in the AMFR by the former is cancelled by the rise by the latter, leading to the negligence of the marital fertility decline if we are not careful about the MAM change, which seems to be the case in recent East Asian societies. If we find the change over the period in average age at marriage (MAM), we should realize that it may be caused by the quantum change in cohort $m(x)$, too. Also, note that the AMFR increase by marriage age change (B4) is limited to controlled fertility as shown in Section 2 (in contrast, for a simulation of this case for natural fertility, see Appendix A).

4 Discussions

4.1 AMFR increase in low fertility societies

There have been a good many societies where marriage is one of the decisive demographic factors of fertility because of the scarcity of births outside marriage. In such societies, analyses of fertility decline by aggregate measures have often been conducted by using AMFR. Recently among these, increases in the AMFR have been reported in many East Asian societies with a remarkable increase in age at marriage, for example, in South Korea (Eun, 2003; Jun, 2003), Hong Kong (Yip and Lee, 2002), Taiwan (Freedman et al., 1994; Chang, 2003), and Japan (Ministry of Health and Welfare, 1998; Tsuya and Mason, 1995; Population Association of Japan, 2002) [14].

Hence, the decomposition almost unanimously results in that “decline in nuptiality accounts for almost all the decline in the” fertility (the CBR or the TFR) “and declining marital fertility for none of it” (Freedman et al., 1994). In these analyses of fertility decline we should remember that the marriages and births have been constantly postponed. Under these circumstances, as we have showed in Simulation B4, the decomposition of the TFR decline using the AMFR will inevitably produce misleading results of the rising contribution of the AMFR. Hence, this decomposition results in an exaggeration of the decline or the postponement of the marriage rate and a negligence of the marital fertility decline as the cause of the fertility decline.

Let us take a closer look at the case of Japan. The percentage of illegitimate births in Japan remained at about 1.5 percent in 2001 and all of the births are virtually from married couples. The mean age of mothers at childbearing increased from 27.75 to 29.65 between 1970 and 2000. This is parallel to a marriage age increase for women from 24.2 to 27.0 (IPSS, 2002). According to a decomposition, the decline (-0.71) in the TFR from 1970 to 1995 (2.13 to 1.42) was decomposed into contributions of changes in marriage (proportion married) and marital fertility (AMFR) as -0.75 and +0.05, respectively (Ministry of Health and Welfare, 1998) [15]. From these kinds of results, it is widely believed in Japan that the fertility decline since the middle of the 1970s has been caused almost solely by the postponement of marriages and that marital fertility or the average number of children per couple has not exerted a negative effect on fertility. This recognition has even aroused some objections and doubts about allocating funds to support childrearing (The Wise, 2000).

In contrast to those decompositions, if we use the CMF instead of the AMFR as a cohort marital fertility measure, 30 percent of the recent TFR decline is attributed to (the quantum and tempo in) marital fertility of cohort (Hirosima, 2001). Interestingly, another simulation analysis, using the AMFR for cohort marital fertility, decomposed the fertility decline as attributable predominantly to cohort marriage postponement and none to cohort marital fertility (AMFR) (Takahashi, 2001). These results of two different simulations by cohort variables that are intended to take into account cohort-period translation show that if we represent the cohort marital fertility by the AMFR, the marital fertility doesn’t appear to contribute to the TFR decline. Also, Ogawa and Retherford (1993) stress the decline in marital fertility (though explicitly mainly referring to first birth), by mostly relying on the decomposition of the total fertility rate based on period parity progression ratios (TFR_{ppr}). They attribute the change in TFR_{ppr} to the decline in marital fertility (period parity progression ratios from marriage to births) by 68.8% (from 2.09 to 1.84 in 1973-80) and 30.1% (from 1.84 to 1.70 in 1980-90) (Table 6).

Further, in contrast to the increase in the AMFR by year, the total marital fertility rate, the sum of the duration-specific marital fertility rate, for each year declined from 2.2 in 1980 to 1.9 in 1995 according to a representative fertility survey in Japan (National Fertility Survey in 1997, IPSS, 1998; p.20). The ineffectiveness of decomposition using the AMFR for recent fertility decline in these societies is obvious.

We refer to another misleading decomposition of the TFR using the AMFR without time change but geographical difference, where the difference between the TFR of Tokyo (1.23) and that of all of Japan (1.52) in 1990 (-0.29) was decomposed into differences of proportion married (–0.29) and marital fertility (AMFR) (0.00) (Ishikawa, 1992). However, the interpretation that the marital fertility in Tokyo was as high as the average of overall Japan has been disproved by other analyses (Hirosima and Mita, 1995). According to the analyses, the total marital fertility rate (composed of the marriage duration fertility rate) in Tokyo was 1.876 in 1982-1986, whereas that for all of Japan was 1.997, according to data from the Ninth National Fertility Survey, 1987 in Japan. Furthermore, the marital fertility (ever-born children) of women aged 35-39 in Tokyo (1.637) is lower than that for all of Japan (1.811) by 0.174, according to the own children tabulation in the Population Census of 1990. We should note that the age at first marriage of women in Tokyo was 26.7 years and that it is older than that for all of Japan (25.9 years old) by 0.8 years in 1990 (Vital Statistics of Japan).

4.2 Two-tempo question in fertility transition

Though our model is developed primarily for controlled fertility, it may be more or less helpful for less-controlled fertility because it has a duration-dependent characteristic to some extent, as commented on in Section 2 and suggested for even natural fertility in Figure 1.

Among decompositions of fertility decline in demographic transition, in terms of the TFR or the CBR, there have been many reports of an ***increase*** in the AMFR, where marriage age increases. For example, in Taiwan for 15-19 and 20-24 years old from 1961 to 1970 (Freedman, Hermalin and Sun, 1972), in West Malaysia under 25 between 1960 and 1969 (Cho and Retherford, 1973), in South Korea for "younger married women" from 1960 to 1966 (Coale, Goldman and Cho, 1982 [16]), and in Bangladesh, Nepal and Pakistan for most of the main age groups (15-19 to 35-39) from 1960-64 to 1980-84 (Retherford and Rele, 1989). These increases in the AMFR were conceived by the authors as "a puzzling increase" (Freedman, Hermalin and Sun, 1972), a "rather irregular pattern" (Cho and Retherford, 1973) or a possible "measurement error" (Retherford and Rele, 1989).

The increase in the AMFR at mostly younger ages in addition to the overall fertility decline, can be interpreted as the tempo change in marital fertility ($g(t)$) as presented by Donaldson and Nichols (1978). In fact, the advancement in the tempo of duration-specific marital fertility ($g(t)$) was substantiated by the analyses of survey data and given substantive explanations (Donaldson and Nichols, 1978; Freedman and Casterline, 1982; Rindfuss and Morgan, 1983). As shown in (the reverse of) Simulation A2-A3, the increase in the AMFR can be really caused by the advancement in the tempo of $g(t)$ without an increase in the quantum of $g(t)$.

If one argues, however, that the increase in the AMFR is totally accounted for by this tempo change, it may be an exaggeration. Because another tempo change in the opposite direction (postponement) in the marriage rate ($m(x)$) also increases the AMFR for most ages except the youngest ages as shown in Simulation A4-A5 if the marital fertility has the duration dependent characteristic to a certain extent. This effect must not be overlooked.

The increase in the AMFR at younger ages makes the contribution by decrease in the overall marital fertility small to the fertility decline, which some demographers have found strange. Freedman, Hermalin and Sun (1972) appropriately cast doubt on standardization using the AMFR as exaggerating the effect of marriage change because those who postpone marriage are likely to be those with eventually lower fertility (CMF).

Then, what will be the result in the quantum of age-specific fertility (TFR) by such tempo changes in age-specific marriage rate (+) and duration-specific marital fertility (-)? If the change in the latter is smaller than that in the former, this may be called a catch-up of family formation, resulting in a smaller decline in the TFR. On the contrary, if the TFR increases, the advancement of tempo in $g(t)$ should be characterized more than as a catch-up including even an increase in the quantum of $g(t)$. Therefore, the rise in Malay fertility in the late 1970s and early 1980s seems to have been caused by factors other than "primarily a short-term response to the very sharp rise in age at marriage and (consequent later schedule of family formation)" (Hirschman, 1986, p.179).

Also, in China, an increase in the AMFR from 1984 to 1987 is reported but, in contrast to the cases above, with a marriage age decrease (Zeng et al., 1991). An increase in the AMFR may be caused by a forward shift or an increase in the duration-specific marital fertility rate ($g(t)$) as simulated (B1 and B2) in the last section. Furthermore, as the reverse of Simulation A4-A5 suggests, the advancement in marriage rate, $m(x)$, makes the AMFR decrease against what Zeng et al. (1991) speculated. Thus the change in marital fertility ($g(t)$) (in form of forward shift or increase) may be larger than observed by an increase in the AMFR. If it is a forward shift rather than an increase in the $g(t)$, it may not be a failure in the family planning program.

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Appendix A: Simulation for natural fertility

In natural fertility marital fertility can be modeled basically as an age-specific function $G(x)$ = AMFR as shown in Section 2. By this model, the influence of marriage on marital fertility is clearly shown in formulas (7) and (8) (no influence) but rather that on age-specific fertility rate (AFR) and the TFR is of some importance.

The AFR is a multiplication of $G(x)$ by the APM (age-specific proportion married). Thus the TFR for a cohort or completed fertility (CF) is expressed as follows, using $G(x)$ instead of $g(x\text{-}a)$ in formula (2),

$$CF = \int_0^{\omega} B(x)dx / P = \int_0^{\omega} G(x)\int_0^{x} m(a)dadx$$

This formula shows that CF is swayed by whether proportion married is high or not over the ages where *G(x)* is large. Since marital fertility in natural fertility *G(x)* monotonously decreases with age as shown in Figure 1, the later the age at marriage, the smaller the CF, as shown in Figure 9 (C) . In dynamic simulation for natural fertility, we postulate that the age at marriage suddenly changes from a cohort in the same manner as it does in the dynamic simulations in Section 3. The result in the period TFR looks the same as the case of quantum reduction in $g(t)$ (Simulation B1, Figure 5) or in $m(x)$ (Simulation B3, Figure 7) because no shift at all occurred in the AFR over later ages. But the temporary decrease in the total marriage rate is the same as Simulation B4 in Figure 8. This means that tempo change in $m(x)$ causes demographic translation in the total marriage rate but not in the TFR for natural fertility. Trussell, Menken and Coale (1982; Figure 5) shows the same decrease in the TFR depicted with a more controlled fertility which approaches the case with marriage delay of Simulation B4.

The values of *G(x)* are set as 0, 0, 1, 1, 0.9, 0.8, 0.75, 0.7, 0.5, 0.1 for age 15 to 26 (for corresponding real ages, see note 11), referring to "average" natural fertility in Henry (1961), and setting $m(x)$ the same as in Simulation B4.

Appendix B: Simulation for CBR

We have examined the analysis by decomposition of only TFR change in Section 3.2. As the relationships, however, of the AMFR to the CMF shown in Section 2 and 3.1 are the same in any kind of analyses, the result referring to decomposition of the TFR can also be relevant to any analyses using the age-specific fertility rate (AFR) and the AMFR. Why? We show the reason using the case of the CBR.

The CBR can be expressed as follows.

$$\text{CBR} = \sum_x B(x)/N = \sum_x \frac{B(x)}{P(x)} \frac{P(x)}{\sum_x P(x)} \frac{\sum_x P(x)}{N}$$

where N denotes total population and x, reproductive age.

This equation shows that the CBR is a kind of weighted total of the AFR, similar to the TFR where the weights are all unity. Thus if the weights are all constant in time, demographic translation holds for the CBR, too, as an extension for the TFR. In reality the weight may change in time, and the effect can be modified. The modifier or weight is the proportion of women in reproductive ages and the proportion of reproductive age women to population. In certain extreme conditions, these weights may cancel out the effect by demographic translation. Hence, since the CBR for cohort is hardly meaningful, we actually argue that the change in the TFR (completed fertility) or in the AFR for cohort affects the (period) CBR by cohort-period translation.

Further, the discussion above holds for simulations of the CBR with the AMFR and the APM as well, if we divide the AFR into these two elements, since the AMFR and the APM have no direct relation with weights shown above,

Appendix C: Distortion in I_g and I_g ' by marriage age change

We examine the marital fertility index, I_g by Coale (1967) because it is based on age-specific rates including the AMFR and has been used as an important measure in many historical and contemporary studies (see, for example, articles in Ruzicka, 1982; Coale and Watkins, 1986).

It is defined as follows (Coale, 1967).

$$I_g = \frac{\sum f_i M_i}{\sum h_i M_i}$$

where h_i denotes the fertility of married Hutterite women in each five-year age interval in the reproductive span, f_i, the observed fertility rate of married women (AMFR) in each age interval in the specified population, and M_i, the number of married women in the age interval. (From M_i , age-specific marriage rate $m(x)$ can be estimated.) This can be expressed as

$$I_g = \frac{B}{\sum h_i (M_i / P_i) P_i}$$

where P_i denotes number of women in age interval i. We can express the numerator as $B = \Sigma f_i M_i$, a sum of products of the AMFR by M_i, and apply different tempos in $g(t)$ producing a different AMFR in I_g. This procedure obviously does not affect B and hence I_g at all. Thus I_g has the virtue of not being affected by tempo in $g(t)$.

Then how about tempo in the age-specific marriage rate, $m(x)$? The conclusion is that I_g is really affected by the change in tempo of $m(x)$. This seems to have been known by demographers (Freedman and Casterline, 1982; Guinnane et al. , 1994). But their arguments are not convincing.

As we see in the formula above, the numerator is constant, B. The denominator is a sum of M_i weighted by h_i. The weight, h_i has a declining shape after age 20-24. If we express M_i as the product of proportion married (M_i / P_i) ($M(x)$ in our text) and P_i , then the denominator is expressed as a sum of proportion married weighted by $h_i P_i$. P_i has the tapering shape unless the population is decreasing with very low fertility and it only accelerates the declining shape of h_i. For convenience hereafter, we assume P_i as constant at any age interval because it only accelerates the effect we discuss. Then we can regard M_i as proportion married as in the text ($M(x)$). Hence if the tempo of M_i delayed then the integral becomes small and I_g becomes higher for the same marital fertility, f_i. Thus, we conclude that the change in age at marriage and its effect on I_g are positively correlated, i. e. if age at marriage increases, I_g increases for the same f_i. It looks like the property of the AMFR in controlled fertility, though it has no direct connection with the AMFR. We can refer to this problem as the marriage age problem too. In this sense, we may be able to say that I_g indirectly involves the AMFR in connection with I_g *’* which, we will show, directly uses the AMFR.

Coale and Tready (1986; p.161), however, argue just the opposite. “Clearly, when fertility is highly controlled, I_g is higher with a given age schedule of marital fertility in an early marrying population than in a late marrying population”. This assertion seems not to be based on calculation and the error seemingly happened from another expression of I_g in the very complicated form shown in their text.

In fact, I_g in Sweden in 1950-60 would be 1.49 times higher had the age schedule of marital fertility of 1950-60 occurred with an age distribution of married women the same as in 1900-10 (late marrying population, not early marrying population), calculated using nuptiality rates in Bogue (1969), Table 17-6.

Also, note that the explanation shown above should not be limited to controlled fertility.

At the same time, the index of proportion married, I_m will be skewed in the opposite direction. I_m is defined as follows (Coale, 1967).

$$I_m = \frac{\sum h_i M_i}{\sum h_i P_i}$$

This can be expressed as

$$I_m = \frac{\sum h_i (M_i / P_i)(P_i / \sum P_i)}{\sum h_i (P_i / \sum P_i)}$$

As shown in the formula, the numerator is the sum of products of h_i (Hutterite fertility rate), the proportion married and the proportion of population by age interval. As we argue on the decreasing shape of h_i, it is convenient to assume P_i is constant at any age interval since the proportion of population by age interval only accelerates the decreasing shape of h_i. If we assume P_i is constant at any age interval, then we can regard M_i as the proportion married as in the case of I_g.

The numerator and denominator are weighted sums. If there were no weight, it would be not affected by the distribution of M_i or P_i . The distribution of h_i has a declining shape as aging so that the later marriage has a negative effect in the numerator. In spite of the same quantum of marriage rate, the tempo of marriage rate affects the index I_m in the opposite direction. Hence, when marriage is being postponed, the decline of proportion married by I_m is exaggerated and marital fertility decline by I_g is underestimated.

To avoid the arbitrariness of M_i in I_g, I_g *'* based upon direct standardization is proposed (Knodel, 1986) as $I_g' = \frac{\sum f_i S_i}{\sum h_i S_i}$ where S_i is the number of married women in the standard population. The influence of marriage age is avoided by using the same married women population of S_i for I_g *'*. Then the influence of marriage age on f_i (AMFR) discussed in this paper is introduced. Actually, I_g *'* is a weighted sum of the AMFR as shown below. Thus it has the same problem with the AMFR shown in this paper.

$$I_g' = \sum f_i \frac{S_i}{\sum S_i} \frac{\sum S_i}{\sum h_i S_i}$$

Hence, we can conclude that the marriage age problem cannot be avoided in using I_g or I_g *'* because of its indirect or direct use of AMFR. Nevertheless, if the fertility is natural fertility, I_g *'* can express the quantum of marital fertility rate as it is not affected by marriage age, as stated in Section 2.2. If we want to detect the onset of demographic transition, however, I_g *'* has the same problem with I_g.

That I_g is affected by marriage age notwithstanding the fertility regime may cause a serious problem. When marriage is stable or getting later, if we detect the decline in I_g, we can interpret it as the decline of $g(t)$ because marriage change in this case does not deflate I_g. But when marriage is getting later, which is very likely in demographic transition, it is very difficult not to miss the decline in marital fertility because the delay in marriage inflates I_g. In this situation, we will find the increase of marital fertility even if the $g(t)$ does not change or even decline to a certain degree. Also, we may find the decline later than the reality. Fortunately it seems not to be the case in the West because "the demographic transition in the

West resulted mainly from a decline in marital fertility, while nuptiality levels were relatively stable" (Freedman, 1982). Obviously, this was not the case in Asian countries (Coale, Goldman and Cho, 1982; Freedman and Casterline, 1982; Kobayashi, 1982). For example, in Japan, the contention that "the proportion married was falling before the major decline in marital fertility occurred" in the demographic transition (ca 1920-1960) measured by I_g and I_m (Tsubouchi, 1970) may be reexamined to know to what extent the decline in marital fertility was really delayed.

Appendix D: Derivation of total effect on quantum of period rate by tempo change in cohort rate

The theorem about the tempo change by real number can be proved in two ways.

Proof 1. Let us take 1/*H* years as a unit of the Lexis diagram and a change of one unit (1/*H*) from the first unit of a year from a cohort by the unit (instead of a one year change on the Lexis diagram of a one year unit, as explained in the text). Think, for example, of a diagram with grids by month and a one-month delay from the first cohort by month of a year (*H*=12). Then the diagram will look almost similar to that shown in Table 3, filled with many more figures by month. Then, the tempo change for a unit (month) change for cohort (by month) obviously results in the total change of *T* because the TFR for a cohort is represented by *T*. But the weight of a (month) cohort in a year is 1/*H*, so that the total change in quantum of period rate is $T*1/H = T/H$ over the period of reproduction starting from the beginning of the change. QED.

Proof 2. We discuss the question using as an illustrative example, the change in marriage rate in Table 3. We set the age-specific marriage rate $m(x)$ for cohorts until 1984 and after 1985 (at age 15) as a_i and b_i, respectively, where i=1,2, 3, …,10, and age $x=14+i$. We set life time cohort marriage rate as $T=\Sigma a_i = \Sigma b_i$, and the difference between the two average ages of marriage rate as *d*. In Table 3, *d* is set as 1 year. But now we set it as a certain decimal, positive or negative. Thus b_i should be different from the values shown in Table 3 to satisfy the condition mentioned above.

As average ages of a_i and b_i are expressed by $\Sigma ia_i / \Sigma a_i$ +14.5 and $\Sigma ib_i / \Sigma b_i$+14.5, respectively, then $d=\Sigma ib_i / \Sigma b_i - \Sigma ia_i / \Sigma a_i$ or $dT=\Sigma ib_i - \Sigma ia_i$… ………..………… …………(a)

The total marriage rate for each year in the period that is influenced by the tempo change in the cohort rate is expressed as follows;

1985: $a_2+a_3+a_4+a_5+a_6+a_7+a_8+a_9+a_{10} \quad +b_1 \quad =T+(a_2-b_2)+(a_3-b_3)+(a_4-b_4)+\ldots+(a_{10}-b_{10})$

1986: $a_3+a_4+a_5+a_6+a_7+a_8+a_9+a_{10} \quad +b_1+b_2 \quad =T+(a_3-b_3)+(a_4-b_4)+(a_5-b_5)+\ldots+(a_{10}-b_{10})$

1987: $a_4+a_5+a_6+a_7+a_8+a_9+a_{10} \quad +b_1+b_2+b_3 \quad =T+(a_4-b_4)+(a_5-b_5)+(a_6-b_6)+\ldots+(a_{10}-b_{10})$

…

1991: $a_8+a_9+a_{10} \quad +b_1+b_2+b_3+b_4+b_5+b_6+b_7=T+(a_8-b_8)+(a_9-b_9)+(a_{10}-b_{10})$

1992: $a_9+a_{10} \quad +b_1+b_2+b_3+b_4+b_5+b_6+b_7+b_8=T+(a_9-b_9)+(a_{10}-b_{10})$

1993: $a_{10} \quad +b_1+b_2+b_3+b_4+b_5+b_6+b_7+b_8+b_9=T+(a_{10}-b_{10})$,

Note that $b_1=T-b_2-b_3-b_4-b_5-b_6-b_7-b_8-b_9-b_{10}$,

$b_1+b_2= T-b_3-b_4-b_5-b_6-b_7-b_8-b_9-b_{10}$, and so forth.

Then the total of these from 1985 to 1993 is

$9T+a_2-b_2+2(a_3-b_3)+3(a_4-b_4)+\ldots+8(a_9-b_9)+9(a_{10}-b_{10})$

$=9T+a_1-b_1+2(a_2-b_2)+3(a_3-b_3)+\ldots+9(a_9-b_9)+10(a_{10}-b_{10}) - \Sigma a_i + \Sigma b_i$
$=9T + \Sigma i a_i - \Sigma i b_i - \Sigma a_i + \Sigma b_i$
$=9T-dT$ (substituting by dT using formula (a)) QED.
Note that the figure, 9, comes from the span of rates (for birth, the period of reproduction, 35 years in reality).
This result means that the total amount of loss caused by the delay, d, of cohort rates over the specific period (e.g. reproductive period) is dT. This applies even if d is negative as noted above.

Appendix E: Tempo change in period caused by quantum change in cohort

We take Simulation B1 (Figure 5) for explanation. The eventual change in the mean age at birth is caused by the change in composition of births by order. Thus if we calculate the mean age at birth by birth order in each year, the eventual change in the mean age at birth will not appear. But the temporal change (rise) in the mean age at birth in about 1990-91 is caused by the compositional change in births by cohort (decrease in births of younger cohorts), not by the compositional change in order-specific birth rate. Hence the rise does appear even if the mean age at birth is calculated by birth order. Such procedure by birth order proposed by Bongaarts and Feeney (1998) ignores this effect and assumes the change in tempo of period fertility as being only caused by a change in tempo of cohort fertility. Even a pure quantum change (decrease) in cohort causes a temporary tempo change (delay) in period as Wunsch and Termote (1978) writes, “synthetic period tempo measures do not usually correspond to true cohort measures, as they are also influenced by cohort intensities”. Note that the influence is opposite or negative, meaning that the decrease in any order birth rate for cohort causes a temporary rise, not drop, in the average age at birth in period as in this case and increase in birth rate for cohort causes its temporal drop. (Ryder (1983), Keilman (2001) and Suzuki (2002) give similar relations in quantitative terms in continuous changes.)

If we adjust the TFR by the delay in period derived from the cohort quantum decrease, as proposed by Bongaarts and Feeney (1998), the adjusted quantum will be distorted to be larger and underestimate the decrease in it.

Notes

1) For example, the difference (change) in the TFR ($\sum_x \{B(x)/P(x)\}$) from point *0* to point *t* (ΔTFR) is decomposed, via the method developed by Kitagawa (1955), as

$\Delta\text{TFR}=\text{TFR}_t\text{-TFR}_0=\sum_x f_t(x)n_t(x) - \sum_x f_0(x)n_0(x)$

$$= \sum_x \{f_t(x) - f_0(x)\}\{n_0(x)+n_t(x)\}/2 + \sum_x \{f_t(x)+f_0(x)\}\{n_t(x)-n_0(x)\}/2$$

where denoted as $f(x)=B(x)/M(x)$ (AMFR), $n(x)=M(x)/P(x)$ (APM), and interpreted the first term as a contribution of difference in marital fertility rate, and the second term as a contribution of difference in proportion married.

2) According to Blake (1985) and Wilson et al. (1988), the AMFR at every age is slightly higher for a marriage cohort that married at later ages because of their shorter duration from

marriage at the same ages in natural fertility, too (see Figure 1). The increases are, however, relatively small compared to the high level of fertility.

3) Tachi (1936) expressed his critical view, possibly among the first in the world, about standardization of the CBR using the AMFR, introducing the discussions through Kuczynski (1935). Against his own remarks, he used the standardization for Japanese fertility in 1920, 1925, and 1930, which may be inappropriate since the fertility was departing from natural fertility by spreading fertility control over the period.

4) For period observation, a fertility measure corresponding to completed marital fertility (CMF) is usually called the total marital fertility rate, which is a sum of the duration-specific marital fertility rate over the duration of marriage in a specific year. This is different from the TMFR defined in formula (4).

5) On the contrary, Ruzicka (1974) postulates the AMFR (*f(m)* by his notation) as the fundamental marital fertility rate as we model for natural fertility in formula (7) (*G(x)*), and analyzes the CMF for Australian cohort 1906-1941 by our formula (9) ($C[a]_{45}$ by his notation). We argue that unless the fertility of these cohorts can be modeled as natural fertility (marital fertility variant by age but constant by marriage age), his arguments may not be justified.

6) This formula corresponds to formula (8) in Trussell (1981) and in Trussell et al. (1982) which includes age (x) and duration (t)-specific marital fertility, $g(x, t)=Tn(x)\exp(-st)$, where $Tn(x)$ denoting age-specific natural fertility level and s, degree of fertility control. This model can neatly express fertility both natural and controlled. By this model, however, it is difficult to express tempo change (or difference) in duration-specific marital fertility without quantum change, which is likely to occur in demographic transition. If fertility-related behavior itself may change in the process of demographic transition in certain societies (Rindfuss and Morgan, 1983), it may be problematic to model such controlled fertility based upon a natural fertility model.

7) For the analysis of period data, we can define the CMF as follows, analogously to formula (5),
CMF=TFR/TFMR,
using the total fertility rate (TFR) and the total first marriage rate (TFMR) because these are equivalent to $\int_0^{\omega} B(x)dx$ and $M(\omega)$, respectively if the vital rates are constant in time.
If we decompose the TFR decline by these period measures, the decomposition may produce better results than that using the AMFR, even if the time constant assumption does not exactly fit the reality (Inaba, 1995), since it does not involve the problem described in this paper.

8) The formula shown in note 6 by Trussell (1981) and Trussell et al. (1982) reduces to $g(x)=Tn(x)$ if we set the degree of fertility control, s=0.

9) In Figure 4 in Section 3, if we give $g(t)$, for example, as (1,1,1) instead of (1,1,0) then the declining curves become convex and more similar to curves for natural fertility in Figure 1.

10) For convenience, we define the rates for all women of a cohort rather than for single population or parity specific population in the cohort. Note that age-specific rates for all (status combined) population of a cohort can be easily converted to occurrence-exposure rate if it is given from the youngest age.

11)If the rates should be more realistic, take the age from 15 to 26 in Table 1 as 0-14, 15-19, 20-22.5, 22.5-24, 25-27.5, 27.5-29, 30-32.5, 32.5-34, 35-37.5, 37.5-39, 40-44, 45-49, respectively.

12) To illustrate the calculation of *B(x)*, let us take *B*(18) as an example, which is the sum of second births of those married at age 16, i.e. 0.05 (marriage rate) multiplied by 1 (marital fertility rate at duration of the second year) and the first birth of married at age 17, 0.10 (marriage rate) multiplied by 1 (marital fertility rate at duration of the first year), yielding 0.15.

13) If we know the tempo changes d_t of a certain range of cohorts (t=1,2,3,…), we can exactly quantify the total amount of change in quantum of period rates over the period of its influence, which will be $\Sigma d_t M_t$, expressing quantum of cohort rates as M_t given no change in the age pattern. (By this formula, the lost fertility rates for the Japanese cohorts born in 1950-1989 was estimated to be 3.91, Hirosima, 2000)

If the same tempo change (*d* years) continues successively for consecutive cohorts with the same quantum of the TMF, then the decrease in quantum measure per year will be approximately TMF-*d*TMR or (1-*d*)TMR which coincides with Ryder's argument (1964), as, needless to say, this discussion holds for the TFR as well, where we exchange the TMR with the TFR, $m(x)$ with $B(x)$, and $M(\omega)$ with the cohort completed fertility.

14) More specifically, rises in the AMFR were reported in South Korea, for women aged 25-29, 30-34 and 35-39 between 1985 and 2000 (Eun, 2003; Jun, 2003), in Hong Kong, for 20-24, 30-34, and 35-39 between 1986 and 1996 (Yip and Lee, 2002), in Taiwan for 15-19 to 35-39 from 1985-2000 (Chang, 2003; except 20-24 from 1983 to 1991, Freedman et al., 1994), and in Japan for 15-19 to 45-49 except 20-24 from 1970 to 2000 (Ministry of Health and Welfare, 1998; Tsuya and Mason, 1995; Population Association of Japan, 2002). Singulate mean age at marriage of women in Korea was 23.2 in 1981 and 27.0 in 2002 (Eun, 2003), and median age at first marriage of women in Hong Kong was 23.9 in 1981 and 27.0 in 2000 (Yip and Lee, 2002). Percentage of births outside marriage in Hong Kong was 1-2 per cent in 1987-1997 (Yip and Lee, 2002).

15) Tsuya and Mason (1995) decomposed the change in the TFR, -0.395 (-100%) from 1975 to 1990, as -149% by marital composition and 49% by marital fertility.

16) Actually, Coale, Goldman and Cho (1982) refers to I_g rather than the AMFR. We include it here due to similar nature of I_g to the AMFR for convenience (see Appendix C).

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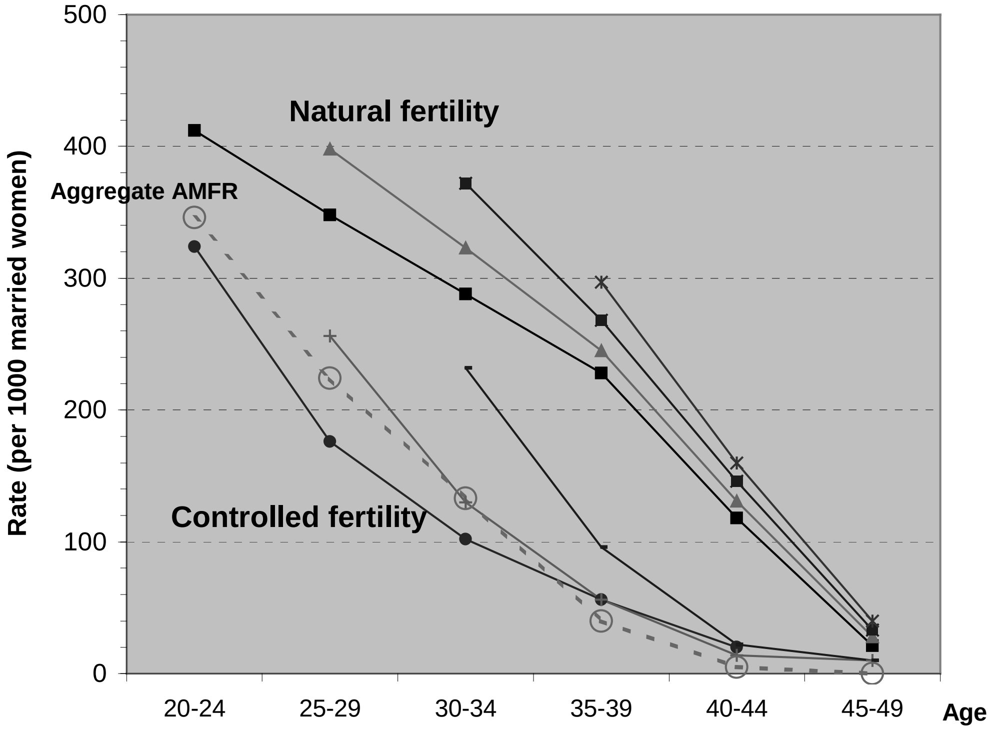
Figure 1. AMFR (age-specific marital fertility rate) for natural fertility and controlled fertilty, by age at marriage, and aggregate AMFR.



The beginning points at the left-hand side of each line show the age at marriage except aggregate AMFR.
Natural fertility: For 14 English parishes, 1600-1799 (Wilson et al. 1988, Figure 8). Married at 15-19 years excluded.
Controlled fertility: Women married at 20-22.5, 25-27.5 and 30-32.5 years in Great Britain in 1919 (Henry, 1976). CMF=3.29, 2.28, and 1.76.
Aggregate AMFR: (Broken line with small circles) Japan, 2000 (Vital Statistics and Population Census).

Table 1 Marriage rates and fertility rates for cohort 15 years old in 1985 (hypothetical data)

Year	Age	AMR	AFR	APM	AMFR
1985	15	0	0	0	-
1986	16	0.05	0	0	-
1987	17	0.10	0.05	0.05	1.00
1988	18	0.15	0.15	0.15	1.00
1989	19	0.20	0.25	0.30	0.83
1990	20	0.20	0.35	0.50	0.70
1991	21	0.15	0.40	0.70	0.57
1992	22	0.10	0.35	0.85	0.41
1993	23	0.05	0.25	0.95	0.26
1994	24	0	0.15	1.00	0.15
1995	25	0	0.05	1.00	0.05
1996	26	0	0.00	1.00	0.00
Total rates		1.0	2.0		4.98
Average age		20.0	21.5		

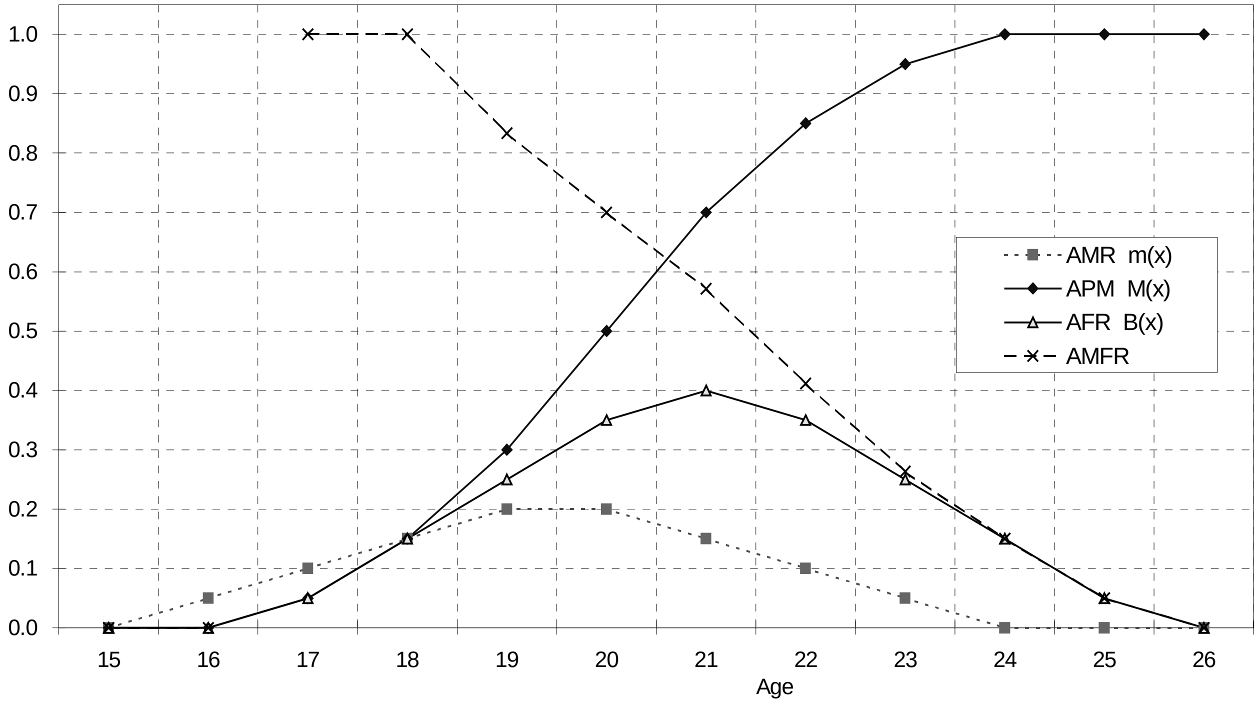
Marriage rates and birth rates are what a cohort aged 15 in 1985 experiences.
AMR: *m(x),* age-specific marriage rates.
AFR*: B(x).* Fertility rates are derived from the marital fertility *g(t)* = (1,1,0). Formula (5).
APM: *M(x).* Proportion married is measured at the beginning of a year. Formula (4).
AMFR: age-specific marital fertility rate.
Total rates are lifetime proportion ever-married, completed fertility (cohort TFR) and TMFR.

Table 2 Duration-specific marital fertility rate

t	*g(t)*
1	1
2	1
3	0

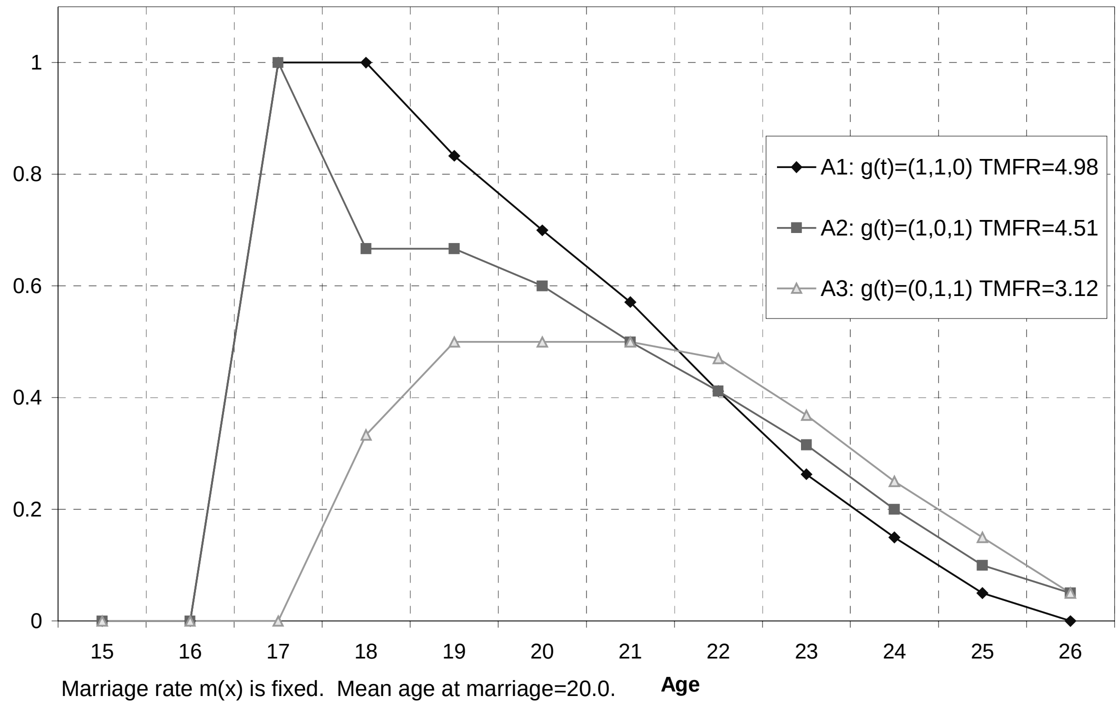
t : marriage duration (year).
g(t): duration-specific marital fertility rate.

Figure 2 Simulated AMFR: Simulation A1: Baseline



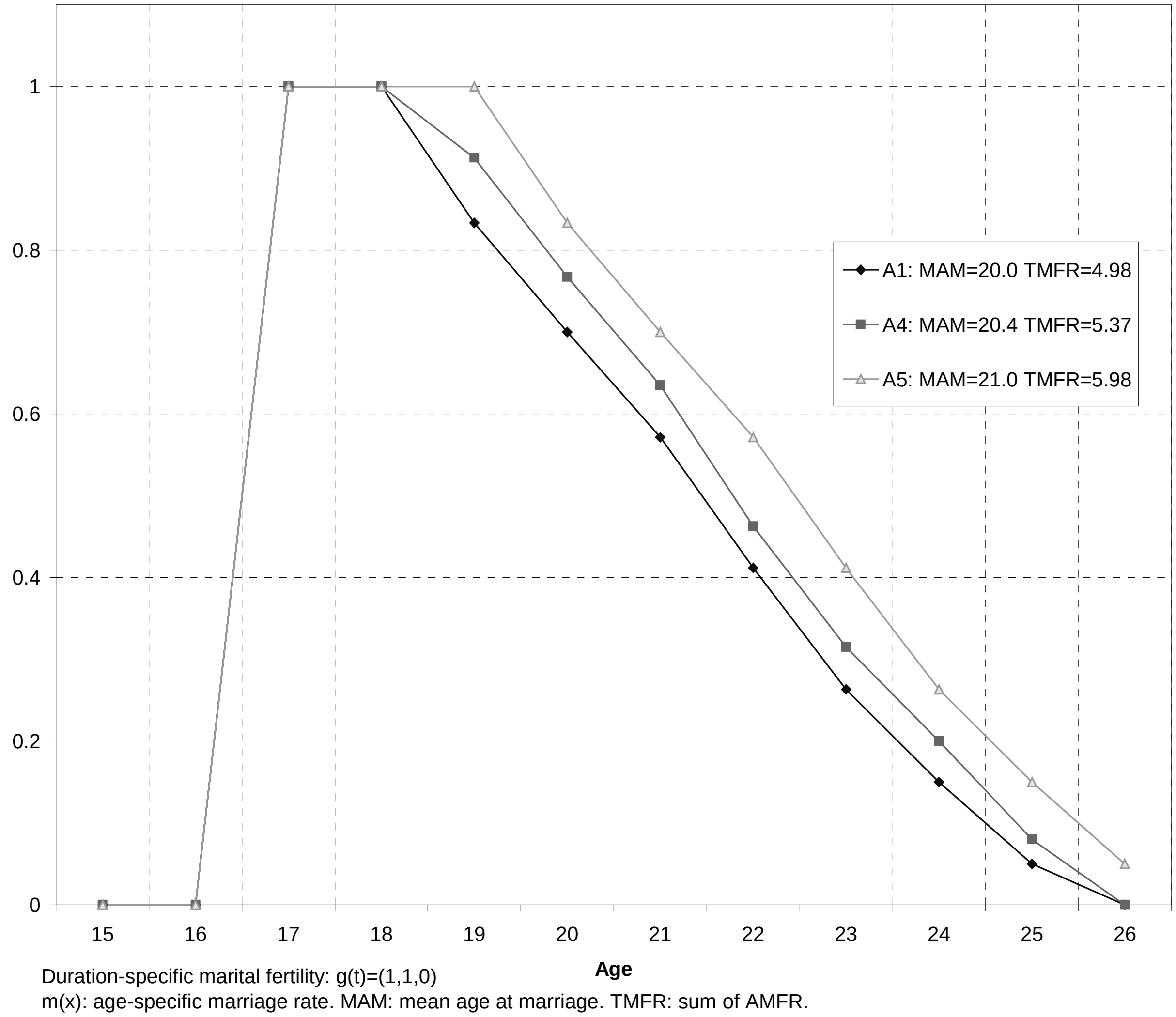
Duration-specific marital fertility rate is g(t)= (1,1,0). Mean age at marriage=20.0.

Figure 3 Simulated AMFR with different tempo in g(t) : Simulation A1, A2, A3



Marriage rate m(x) is fixed. Mean age at marriage=20.0.
g(t): duration-specific marital fertility rate.

Figure 4 Simulated AMFR with different tempo in m(x): Simulation A1, A4, A5



Duration-specific marital fertility: g(t)=(1,1,0)
m(x): age-specific marriage rate. MAM: mean age at marriage. TMFR: sum of AMFR.

Table3 Simulation B4: Calculation of rates (AMR and ABR)

Age	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996
AMR (age-specific marriage rate)																
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0.05	0.05	0.05	0.05	0.05	0	0	0	0	0	0	0	0	0	0	0
17	0.10	0.10	0.10	0.10	0.10	0.10	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
18	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
19	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15
20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20
21	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.20	0.20	0.20	0.20	0.20	0.20
22	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.15	0.15	0.15	0.15	0.15
23		0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.10	0.10	0.10	0.10
24			0	0	0	0	0	0	0	0	0	0	0	0.05	0.05	0.05
25				0	0	0	0	0	0	0	0	0	0	0	0	0
26					0	0	0	0	0	0	0	0	0	0	0	0
MAM (mean age at marriage)					20	20.2	20.3	20.4	20.5	20.5	20.6	20.7	20.8	21	21	21
Total marriage rate					1.00	0.95	0.90	0.85	0.80	0.80	0.85	0.90	0.95	1.00	1.00	1.00
AFR (age-specific fertility rate)																
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17			0.05	0.05	0.05	0.05	0	0	0	0	0	0	0	0	0	0
18			0.15	0.15	0.15	0.15	0.15	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
19			0.25	0.25	0.25	0.25	0.25	0.25	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15
20			0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.25	0.25	0.25	0.25	0.25	0.25	0.25
21			0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.35	0.35	0.35	0.35	0.35	0.35
22			0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.4	0.4	0.4	0.4	0.4
23			0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.35	0.35	0.35	0.35
24			0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.25	0.25	0.25
25				0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.15	0.15
26					0	0	0	0	0	0	0	0	0	0	0	0.05
MAB (mean age at birth)					21.5	21.5	21.6	21.8	21.9	22	22	22	22.1	22.2	22.4	22.5
TFR					2.00	2.00	1.95	1.85	1.75	1.65	1.6	1.65	1.75	1.85	1.95	2.00

The shade shows the cohort aged 15 years in 1985.

Marriage rates are delayed by 1 year beginning with that cohort.

Table4 Simulation B4 (cont'd): Calculation of rates (APM and AMFR)

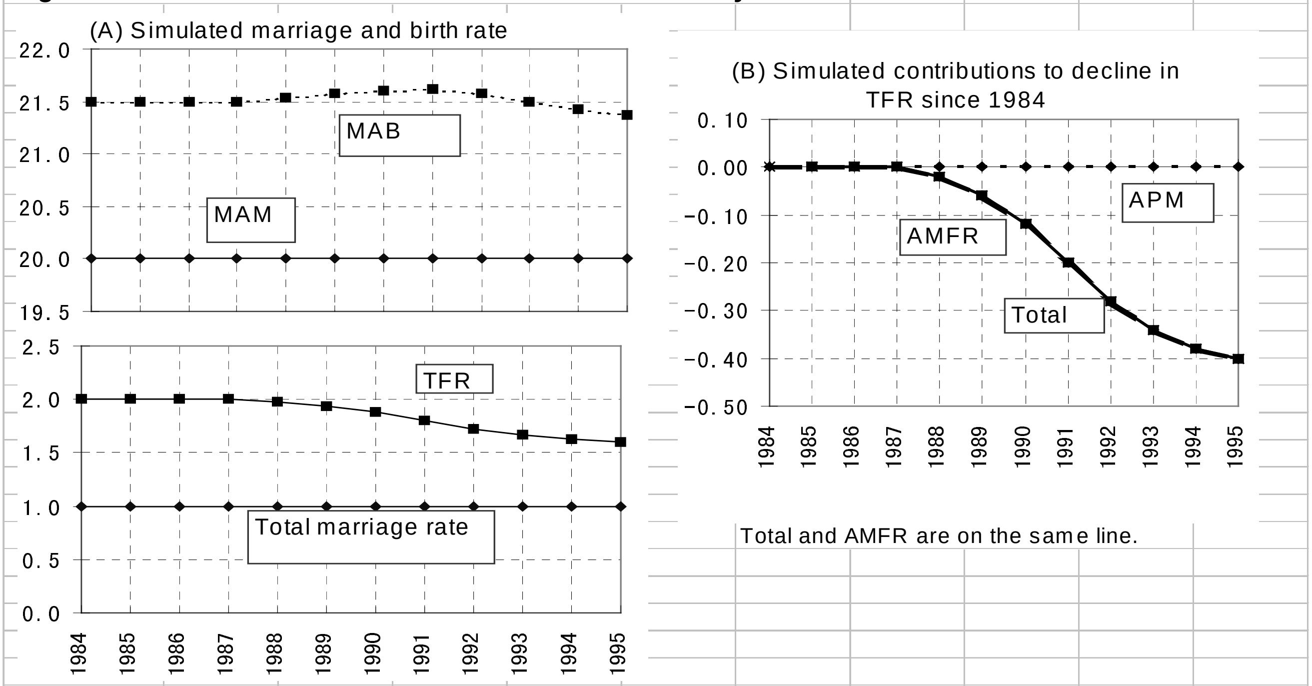
Age	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996
APM (age-specific proportion married)												
15	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0
17	0.05	0.05	0	0	0	0	0	0	0	0	0	0
18	0.15	0.15	0.15	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
19	0.3	0.3	0.3	0.3	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15
20	0.5	0.5	0.5	0.5	0.5	0.3	0.3	0.3	0.3	0.3	0.3	0.3
21	0.7	0.7	0.7	0.7	0.7	0.7	0.5	0.5	0.5	0.5	0.5	0.5
22	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.7	0.7	0.7	0.7	0.7
23	0.95	0.95	0.95	0.95	0.95	0.95	0.95	0.95	0.85	0.85	0.85	0.85
24	1	1	1	1	1	1	1	1	1	0.95	0.95	0.95
25	1	1	1	1	1	1	1	1	1	1	1	1
26	1	1	1	1	1	1	1	1	1	1	1	1
AMFR (age-specific marital fertility rate)												
15	-	-	-	-	-	-	-	-	-	-	-	-
16	-	-	-	-	-	-	-	-	-	-	-	-
17	1	1	1	1	1	1	1	1	1	1	1	1
18	1	1	1	1	1	1	1	1	1	1	1	1
19	0.833	0.833	0.833	0.833	1	1	1	1	1	1	1	1
20	0.7	0.7	0.7	0.7	0.7	0.833	0.833	0.833	0.833	0.833	0.833	0.833
21	0.571	0.571	0.571	0.571	0.571	0.571	0.7	0.7	0.7	0.7	0.7	0.7
22	0.412	0.412	0.412	0.412	0.412	0.412	0.412	0.571	0.571	0.571	0.571	0.571
23	0.263	0.263	0.263	0.263	0.263	0.263	0.263	0.263	0.412	0.412	0.412	0.412
24	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.263	0.263	0.263
25	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.15	0.15
26	0	0	0	0	0	0	0	0	0	0	0	0.05
TMFR	4.98	4.98	4.98	4.98	5.15	5.28	5.41	5.57	5.72	5.83	5.93	5.98

See notes on Table 3.

TMFR (total marital fertility) is the sum of AMFR.

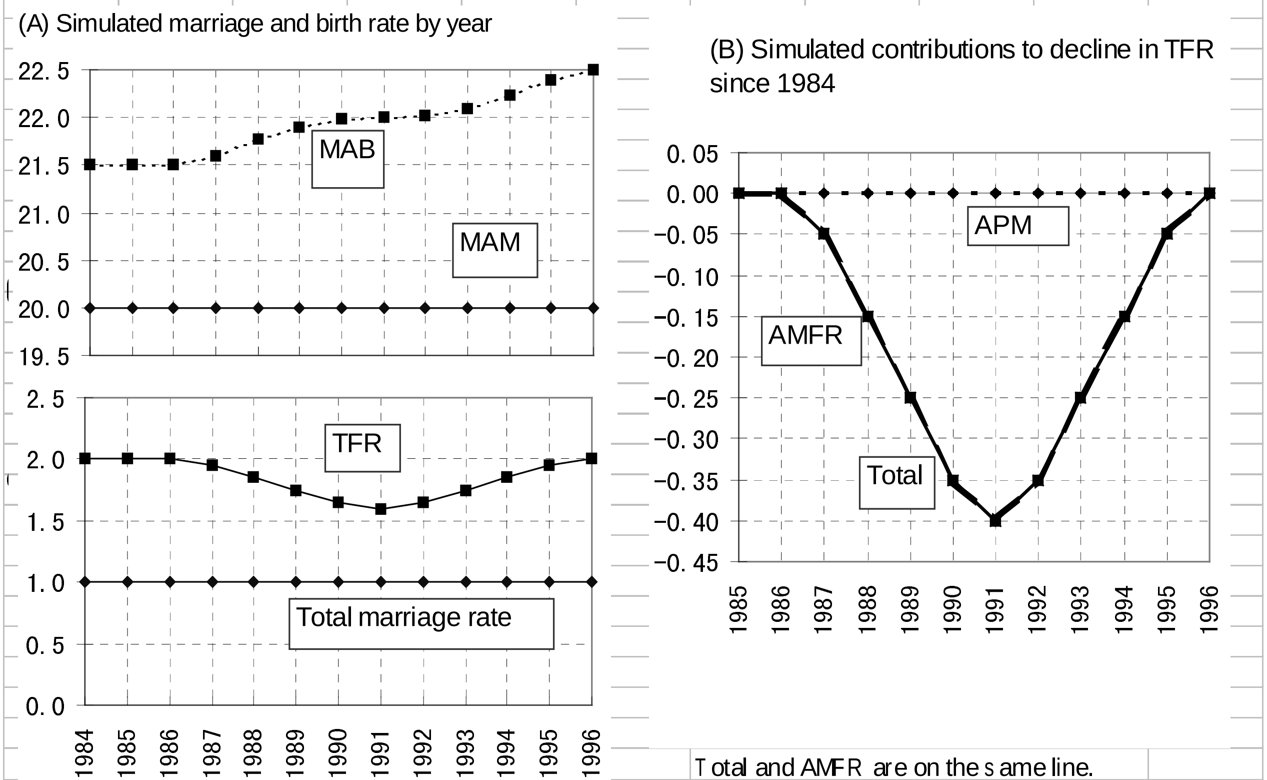
AMFR at 17 is 1 after 1987 because AMR at 15 is set as a very small number instead of 0 after 1986.

Figure 5 Simulation B1: Decline of marital fertility.



Decreased marital fertility from the cohort 15 years old in 1985: g(t)=(1,1,0) ->(1,0.6,0).

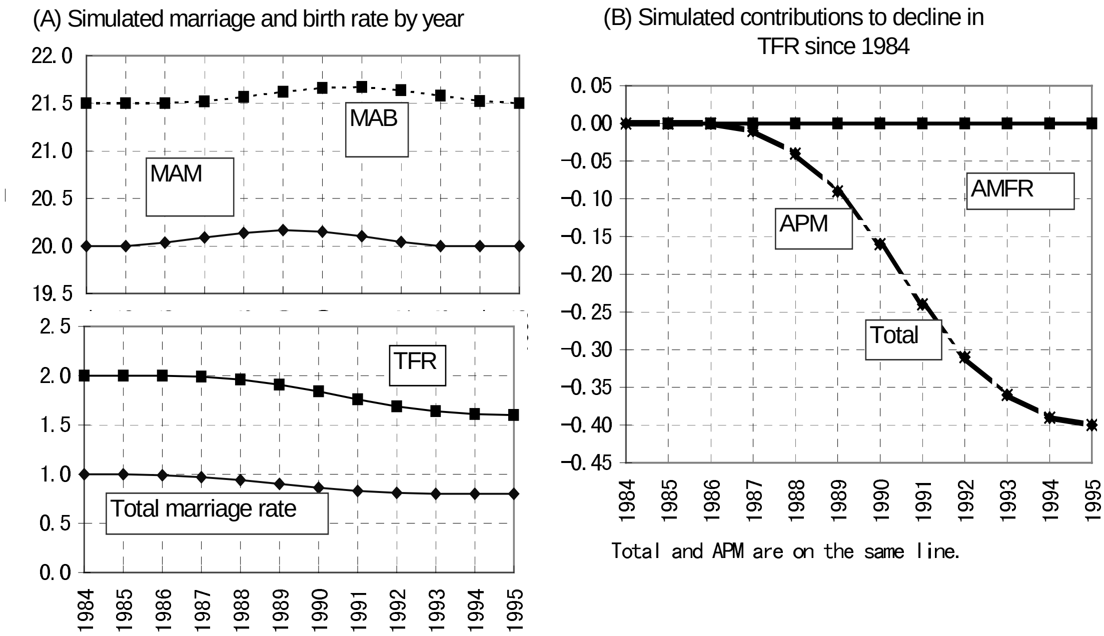
Figure 6 Simulation B2: Postponement of marital fertility.



Total and AMFR are on the same line.

Delayed marital fertility from the cohort 15 years old in 1985: g(t)=(1,1,0) ->(0, 1, 1).

Figure 7 Simulation B3: Decline of marriage rate

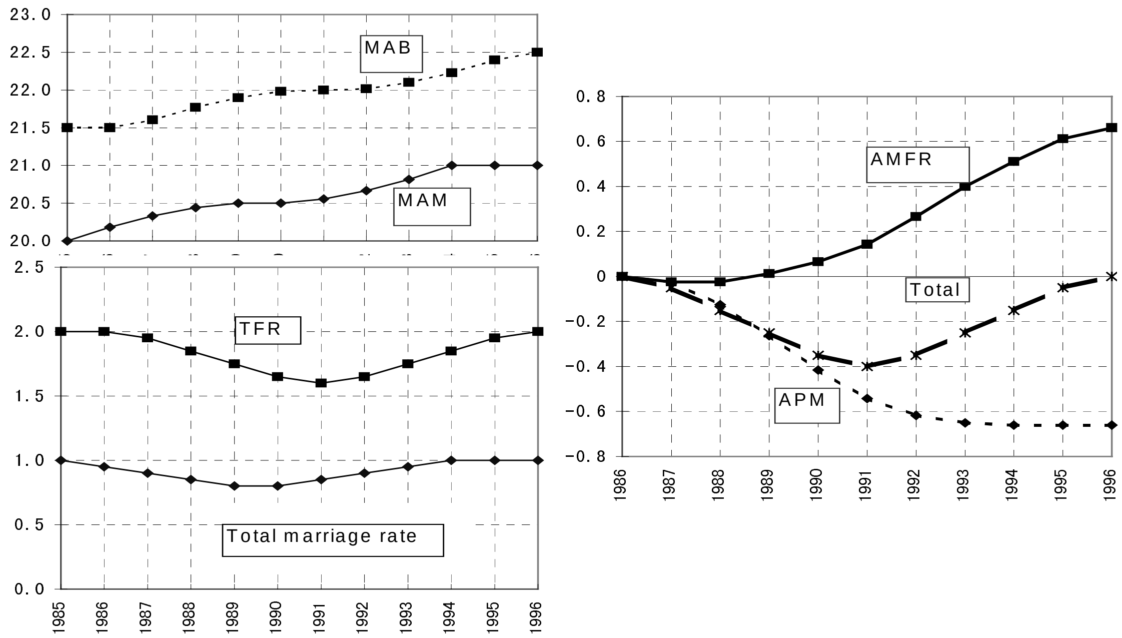


Decreased marriage rate from the cohort 15 years old in 1985 (lifetime ever-married rate: 1 ->0.8).

Figure 8 Simulation B4: Postponement of marriage rate.

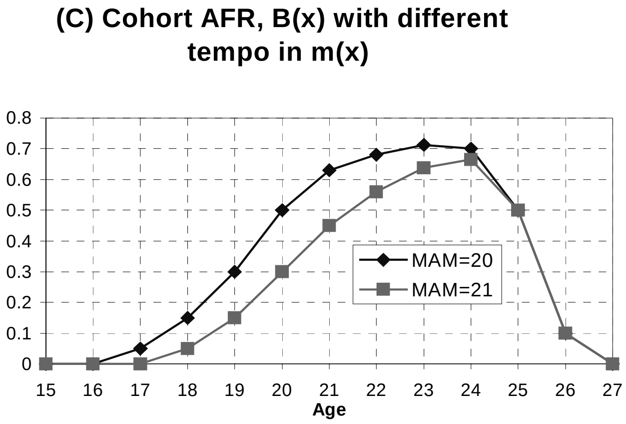
(A) Simulated marriage and birth rate by year

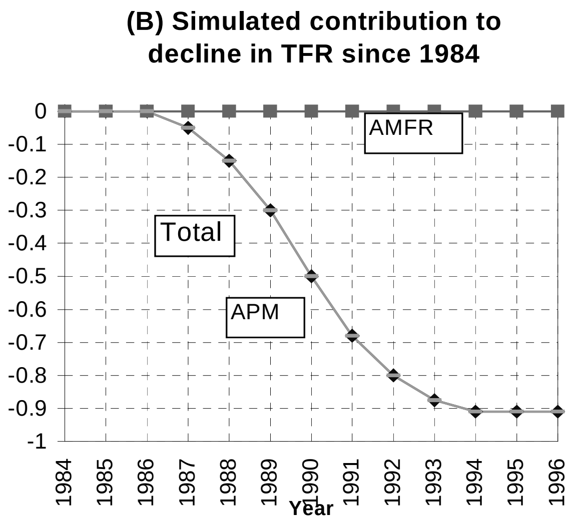
(B) Simulated



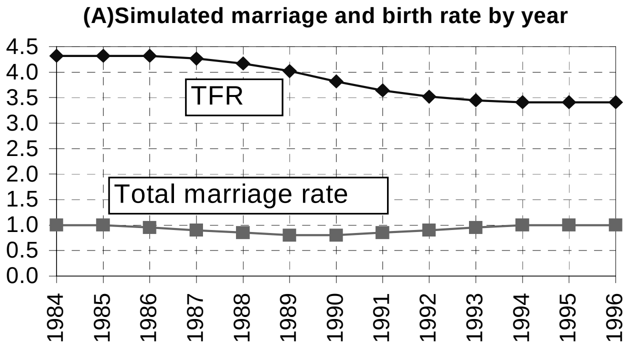
One year delayed marriage rate from the cohort 15 years old in 1985 (mean age at marriage:20.0 -> 21.0 yr.) .

Figure 9 Simulation for natural fertility: postponement of marriage





Total and APM are on the same line.



One year delayed marriage rate (MAM=21.0) from cohort 15 y. in 1985 with constant AMFR in time.