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How Mortality Improvement Increases Population Growth

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How Mortality Improvement Increases Population Growth

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1 Introduction

Pseudo-stable and quasi-stable population models were developed by Coale [3], Feichtinger [6], Dinkel [5] and others. Building on an article by Bennett and Horiuchi [2], Preston and Coale [12], Arthur and Vaupel [1], and Kim [10] further extended these models to develop general relationships for arbitrary population surfaces over age and time. In the spirit of these demographers, we present and prove in this article some formulas that capture the impact of mortality improvement on population growth. Goldstein and Schlag [7] also investigate this question but from a different perspective.

It is sometimes forgotten that in addition to babies and immigrants, people whose lives are saved also augment populations. Over the past century, most of the momentous increase in the world's population has been fueled by increased survival. Birth rates have tended to fall, often sharply, but death rates have decreased even faster. As a result, the population of the world has multiplied. In the future, global birth rates may fall below two children per woman, as they have already done in Europe and parts of the Far East. Population declines caused by such low fertility will be somewhat offset by mortality improvements, to the extent such improvements continue and are substantial. Even without net in-migration, a country's "replacement level of fertility" may be less than 2.0 if lifespans continue to lengthen.

Consider a population closed to migration with a continuous population surface $N(x, y)$ over age x and time y . See Keiding [9] and Arthur and Vaupel

[1] for a discussion of this fundamental but elusive quantity, which is often referred to as age-specific population size. Let the intensity of population growth be denoted by

$$\rho(x, y) = \frac{dN(x, y)/dy}{N(x, y)} = \acute{N}(x, y), \quad (1)$$

where the acute accent, here and elsewhere in this article, denotes the relative derivative with respect to time. Let the intensity of population growth for the population as a whole be denoted by

$$\bar{\rho}(y) = \frac{\int_0^\omega \rho(x, y)N(x, y)dx}{\int_0^\omega N(x, y)dx}, \quad (2)$$

where ω is the highest age attained. Letting

$$T(y) = \int_0^\omega N(x, y)dx, \quad (3)$$

be the total population, note that

$$\bar{\rho}(y) = \acute{T}(y) = \frac{dT(y)/dy}{T(y)}. \quad (4)$$

The quantity $\bar{\rho}(y)$, which is often called the population growth rate, is of prime interest to us.

Let the intensity of mortality (also known as the hazard of death or force of mortality) be given by

$$\mu(x, y) = -\frac{dN(x + a, y + a)/da}{N(x, y)} \quad (5)$$

and let the intensity of improvement in mortality be denoted by

$$\acute{\mu}(x, y) = -\frac{d\mu(x, y)/dy}{\mu(x, y)}. \quad (6)$$

Our focus is on how $\acute{\mu}(x, y)$ affects $\bar{\rho}(y)$.

As shown by Preston and Coale [12],

$$N(x, y) = B(y)s(x, y)R(x, y), \quad (7)$$

where $B(y)$ denotes the number of births at time y , $s(x, y)$ is the period survival function,

$$s(x, y) = e^{-\int_0^x \mu(a, y) da} \quad (8)$$

and

$$R(x, y) = e^{-\int_0^x \rho(a, y) da}. \quad (9)$$

As emphasised by Arthur and Vaupel [1],

$$N(x, y) = B(y - x) s_c(x, y), \quad (10)$$

where $B(y - x)$ denotes the number of births at time $y - x$ and $s_c(x, y)$ is the cohort survival function

$$s_c(x, y) = e^{-\int_0^x \mu(a, y-x+a) da}. \quad (11)$$

It follows from (7) and (10) that

$$R(x, y) = \frac{B(y - x) s_c(x, y)}{B(y) s(x, y)}, \quad (12)$$

so $R(x, y)$ can be considered to be a cohort-period adjustment that captures the dissimilarity between actual cohort and synthetic period values.

To understand how population growth is related to changes in the number of births, to improvements in mortality, and to the cohort-period adjustment, it makes sense to start with a general result that is useful for a variety of different kinds of decomposition. Consider some total or sum $V(y)$:

$$V(y) = \int v(x, y) dx \quad (13)$$

or

$$V(y) = \sum_x v(x, y). \quad (14)$$

Suppose

$$v(x, y) = u_1(x, y) u_2(x, y) \dots u_n(x, y). \quad (15)$$

Then

$$\dot{V}(y) = \sum_{i=1}^n E [\dot{u}_i(x, y)], \quad (16)$$

where

$$E [\dot{u}_i(x, y)] = \frac{\int \dot{u}_i(x, y)v(x, y)dx}{\int v(x, y)dx} \quad (17)$$

or

$$E [\dot{u}_i(x, y)] = \frac{\sum_x \dot{u}_i(x, y)v(x, y)}{\sum_x v(x, y)}. \quad (18)$$

To prove this result, note that

$$\dot{v}(x, y) = \dot{u}_1(x, y) + \dot{u}_2(x, y) + \dots + \dot{u}_n(x, y) \quad (19)$$

and

$$\dot{V}(y) = \frac{\int \dot{v}(x, y)v(x, y)dx}{\int v(x, y)dx}. \quad (20)$$

The general result of formula (16) can be used in equation (3) after substituting (7) to decompose population growth into three components:

$$\bar{\rho}(y) = \dot{B}(y) + \bar{s}(y) + \bar{R}(y), \quad (21)$$

where $\dot{B}(y)$ is the intensity of change in births, $\bar{s}(y)$ is the average intensity of change in survival,

$$\bar{s}(y) = \frac{\int_0^\omega \frac{ds(x, y)/dy}{s(x, y)} N(x, y)dx}{\int_0^\omega N(x, y)dx} \quad (22)$$

and $\bar{R}(y)$ is the average intensity of change in the cohort-period adjustment

$$\bar{R}(y) = \frac{\int_0^\omega \frac{dR(x, y)/dy}{R(x, y)} N(x, y)dx}{\int_0^\omega N(x, y)dx}. \quad (23)$$

It should be noted, however, that the second term on the right-hand side of (21) reflects both current changes in mortality and historical factors that have determined the current population structure. To eliminate the

influence of past changes in fertility and mortality (and perhaps migration), the following result, due to Vaupel [14], is useful:

$$E_{w_2}(v) - E_{w_1}(v) = \frac{Cov_{w_1}(v, \varphi)}{E_{w_1}(\varphi)}, \quad (24)$$

where

$$\varphi \equiv \varphi(x) = \frac{w_2(x)}{w_1(x)}. \quad (25)$$

Note that expected values are given by

$$E_{w_i}(v) = \frac{\int_0^\omega v(x)w_i(x)dx}{\int_0^\omega w_i(x)dx} \quad (26)$$

and the covariance is given by

$$Cov_{w_1}(v, \varphi) = \frac{\int_0^\omega v(x)\varphi(x)w_1(x)dx}{\int_0^\omega w_1(x)dx} - \frac{\int_0^\omega v(x)w_1(x)dx}{\int_0^\omega w_1(x)dx} \frac{\int_0^\omega \varphi(x)w_1(x)dx}{\int_0^\omega w_1(x)dx}. \quad (27)$$

Formula (24) is readily proved by substitution and simplification.

Results (21) and (24) imply

$$\bar{\rho}(y) = \hat{B}(y) + \acute{e}_o(y) + R^*(y), \quad (28)$$

where

$$R^*(y) = \bar{R}(y) - Cov_N(\acute{s}(x, y), R^{-1}) \frac{T(y)}{B(y)e_o(y)}. \quad (29)$$

To prove this, let $w_1(x) = N(x, y)$ and $w_2(x) = B(y)s(x, y)$ and note that

$$E_{s(x,y)}(\acute{s}(x, y)) = \acute{e}_o(y), \quad (30)$$

because

$$e_o(y) = \int_0^\omega s(x, y)dx. \quad (31)$$

Formula (28) permits decomposition of the current population growth rate into

- (1) the current intensity of change in births,
- (2) the current intensity of change in period life expectancy (which captures the impact of current mortality change), and
- (3) a residual term that reflects the influence of historical fluctuations that have resulted in a population size and structure that is different from the stationary population size and structure implied by current mortality and births.

Note that (7) implies that

$$R(x, y) = \frac{N(x, y)}{B(y)s(x, y)}. \quad (32)$$

Regardless of whether a population is open or closed to migration, the value of $R(x, y)$ can thus be interpreted as the ratio of the actual population to the stationary life-table population. Hence, if the actual population structure is the same as the life-table structure, i.e.,

$$N(x, y) = B(y)s(x, y), \quad (33)$$

then

$$R(x, y) = 1, \text{ for all } x \quad (34)$$

and

$$R^*(y) = 0. \quad (35)$$

Table 1 provides some illustrative examples of the decomposition in (28). The population growth rate was estimated by

$$\bar{\rho}(y) \approx \frac{\ln \left[\frac{T(y+5)}{T(y-5)} \right]}{10}. \quad (36)$$

The intensity of change in births was estimated by

$$\dot{B}(y) \approx \frac{\ln \left[\frac{B(y+5)}{B(y-5)} \right]}{10}. \quad (37)$$

Similarly, the intensity of change in period life expectancy was estimated by

Table 1: Decomposition of Population Growth Rate

	<i>Year</i>	$\bar{\rho}(y)\%$	$\acute{B}(y)\%$	$\acute{e}_o(y)\%$	$R^*(y)\%$
World	1960	1.92	1.40	1.21	-0.69
	1990	1.60	-0.11	0.39	1.32
	2040	0.55	0.07	0.20	0.28
Austria	1990	0.60	0.14	0.39	0.07
Belgium	1990	0.27	0.13	0.30	-0.16
Finland	1990	0.41	0.04	0.28	0.09
Germany	1990	0.48	-0.62	0.30	0.80
Italy	1990	0.12	-1.02	0.30	0.84
United Kingdom	1990	0.33	-0.25	0.26	0.33
USA	1990	0.99	-0.49	0.24	1.24

Source: Data from Eurostat [4]; World and USA from United Nations [13].

$$\acute{e}_o \approx \frac{\ln \left[\frac{e_o(y+5)}{e_o(y-5)} \right]}{10}. \quad (38)$$

Finally, $R^*(y)$ was simply estimated as the residual

$$R^*(y) = \bar{\rho}(y) - \acute{B}(y) - \acute{e}_o(y). \quad (39)$$

In Keyfitz [8] and Vaupel [15] the impact on life expectancy of changes in age-specific death rates is analyzed. Their results, when combined with (28), shed light on how mortality change affects population growth. If the rate of mortality improvement is constant over age,

$$\acute{\mu}(x, y) = \acute{\mu}(y), \text{ for all } x, \quad (40)$$

then Keyfitz [8] shows that

$$\acute{e}_o(y) = \acute{\mu}(y)H(y), \quad (41)$$

where $H(y)$ is given by

$$H(y) = -\frac{\int_0^\omega s(x, y) \ln[s(x, y)] dx}{\int_0^\omega s(x, y) dx} \quad (42)$$

and can be interpreted as the entropy of the survival function. If mortality follows a Gompertz trajectory,

$$\mu(x, y) = a_y e^{bx}, \quad (43)$$

then Vaupel [15] indicates that

$$\acute{e}_o(y) \approx \frac{\acute{\mu}(y)/b}{e_o(y)} \quad (44)$$

and

$$e_o(y+1) - e_o(y) \approx \frac{\acute{\mu}(y+0.5)}{b}. \quad (45)$$

Note that (45) implies that a constant rate of mortality improvement will continue to increase life expectancy by about the same absolute amount. On the other hand, (44) implies that as life expectancy increases, the relative rate (i.e., intensity) of improvement will fall.

Because Vaupel's derivation of these approximations is in an unpublished working paper (Vaupel [16]), we provide a derivation here. Approximation (45) follows from (44) via the approximation

$$\acute{e}_o(y) \approx \frac{e_o(y+1) - e_o(y)}{e_o(y)}. \quad (46)$$

Approximation (44) can be derived from (41) by showing that

$$e_o(y)H(y) \approx \frac{1}{b}. \quad (47)$$

Substituting (43) in (8) and then in (42) and then substituting the left hand side of (43), (8) and (31) yields

$$e_o(y)H(y) = \frac{1}{b} \left[\int_0^\omega \mu(x, y)s(x, y)dx - a_y \int_0^\omega s(x, y)dx \right] \quad (48)$$

$$= \frac{1}{b} [1 - a_y e_o(y)]. \quad (49)$$

If $a_y \ll e_o$, as it generally is in low-mortality populations, then the approximation follows. The approximation gets better as mortality improvements are made, because (44) implies that a_y declines faster than e_o rises.

Consider now the relationship between the total fertility rate (TFR) and the intensity of change in the number of births. Let A be the average age of childbearing. Then the net reproduction rate (NRR) is approximately given by

$$NRR \approx \pi s(A) TFR, \quad (50)$$

where π is the proportion of female births and $s(A)$ is a girl's chance of surviving to the average age of childbearing (Coale [3]). Under current conditions in developed countries, $NRR \approx 0.48 TFR$, so that an NRR of about one will be produced by a TFR of about 2.1. In a stable population (i.e., with fixed age-specific fertility and mortality rates), a well-known result from Lotka [11] implies

$$\dot{B} \approx \frac{NRR - 1}{A}. \quad (51)$$

Assume $R^* = 0$. Let the TFR be 1.99, let life expectancy at birth be 80, and let the pace of mortality improvement be 1.5% per year (which is close to the average current level in some developed countries). Assume mortality increases exponentially at a rate of 0.1. Then

$$\bar{\rho}(y) = \dot{B}(y) + \acute{e}_o(y) = -0.0015 + 0.0019 = 0.0004. \quad (52)$$

Although stylised, this result shows that population growth can be positive even if the TFR is below the so-called replacement level.

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