Ageing, Optimal National Saving and Future Living Standards in Australia

ABSTRACT

By simulating a model of the optimal level of saving in a small open economy, this paper calculates the levels of optimal national saving and future living standards for Australia for the period from 1999 to 2050. The calculations focus on the implications of making allowance for the ageing structure of the population. The effects of the ageing structure on employment participation, labour productivity and consumption demands are allowed for. In the 50 years following 1999 the optimal saving response to the ageing of the Australian population is for national saving to increase by 3.2 per cent of GDP up to the year 2010 and then to decline to the year 2050. The implied growth of consumption per consumption unit, a weighted measure of people that allows for the higher consumption demands of old people, is 1.01 per cent per year. It was found that reduced immigration would reduce the rate of growth of living standards but that reduced fertility would not.

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I. INTRODUCTION

Australia faces a significant ageing of its population in the next three decades. For example the proportion of people aged 65 and over is projected to increase from 12.1 percent in 1999 to 20.1 percent in 2031. This ageing of the population profile will put significant pressure on resources because the number of people in the workforce will decrease relative to the number of people demanding consumption goods and services. In anticipation of this pressure commentators in Australia have called for a significant increase in the level of saving. For example, Fitzgerald (1993) recommended an increase in the rate of national saving of five percentage points of GDP partly to meet the increased pressure on resources from the prospective ageing of the Australian population. Ablett (1996), on the basis of intergenerational accounting, concludes that the rates of saving in the early 1990’s were too low in the sense that they imply an imbalance in consumption in the favour of currently living generations. On the other hand, Cutler et al (1990) argue that the demographic future in the US does not require an increase in the rate of national saving.

1 We thank Jeff Borland, John Creedy, Mark Crosby, Daina McDonald, Peter Stemp and two referees for helpful comments.
2 This figure for 2031 comes from the base case population projection in this paper, see below for details.
3 At the time when Fitzgerald (1993) called for a significant increase in the rate of national saving, the rate was much lower than at present, at around 16 percent of GDP.
4 Cashin and McDermott (1998), on the basis of an econometric analysis of a present value model of the current account, conclude that “the increase in net national saving required to satisfy (Australia’s) external borrowing constraint is about 2 to 4 per cent of GDP”. And this is without allowing for ageing - Cashin and McDermotts approach does not allow for demographic change.
To understand how saving should ideally respond to prospective demographic change, it has to be recognised that saving is a means to redistribute consumption over time. A high level of saving will shift consumption into the future. A low level of saving brings consumption towards the present. The optimal rate of saving balances consumption now against consumption in the future in a way which maximises the total utility enjoyed by all people, that is by those living now and those living in the future. As will be seen below, a major influence on the optimal pattern over time of saving is changing demographic structure. For example, a country facing an ageing population, such as Australia, faces the challenge of satisfying in the future the consumption demands of a population with relatively low productive potential. High saving now will help the future population protect its consumption levels against the pressure of a low productive potential. But high saving now comes with a cost. The cost is lower consumption now. Balancing the consumption levels of the current population against the consumption levels of future population is an example of calculating the optimal rate of saving.

This paper investigates for Australia the optimal response of saving to the ageing profile of its population.\(^5\) The paper uses a model of the socially optimal level of saving for a small open economy developed in previous work, see especially Guest and McDonald (1998a). This model is extended in this paper by incorporating predictions of the future age composition of the population and labour force and by introducing a reference level of consumption into the social welfare function.\(^6\) The population and labour force are weighted to account for the age distribution of consumption demands, labour productivity and employment/population ratios. The paper also takes account of the fact that the age composition of Australia’s future population will be affected by the rate of immigration. To assess the effect on optimal saving of the rate of immigration, the paper considers three population projections based on three different assumptions with regard to immigration into Australia.

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\(^5\) In the literature the paper which is the closest to ours in focussing on the socially optimal level of saving is Cutler et al (1990). There are other papers which focus on the effect of ageing on saving but which do not use the concept of the optimal level of saving to evaluate performance, see eg Yashiro and Oishi (1997)

\(^6\) The inclusion of reference consumption in the social welfare function is in keeping with the idea, as expressed by Kahneman, Knetsch and Thaler (1991, p.199), for example that “the significant carriers of utility are…changes relative to a neutral reference point”. In our specification of this idea adopted in this paper, the neutral reference point is consumption in the previous period. Given the widespread support for the importance of reference levels in determining people’s utility, it is of value to consider the impact of this on optimal consumption plans. The effect of this specification on the levels of optimal national saving is discussed in Section V.
In calculating the optimal pattern of saving, the model can also recover the associated levels of consumption per person through the time horizon. These levels of consumption, identified as living standards, are also reported in the paper. They are, within the confines of the model, the best possible outcomes for living standards, where “best” is the living standard which maximises the social welfare function.

Some warnings to the reader are appropriate at the outset in order to clarify the interpretation of the results. First we stress that we calculate optimal saving. To fall short of the optimal level of saving does not necessarily imply disaster. Second, while this paper focuses on the optimal response of saving to an ageing population, it is recognised that there are other mechanisms, in addition to saving, through which society can and probably should use to deal with the pressure on resources caused by an ageing population. For example one aspect of the resource pressure is a significant increase, on current policies, of government outlays, see Guest and McDonald (2000). Part of the optimal response to this would probably involve a shift to user pays for services currently provided by the government. Such a shift will reduce the taxation burden and thus the deadweight cost of government outlays. It may also reduce wasteful excessiveness in the level of provision of these services. Another mechanism to deal with the resource pressure caused by an ageing population is to raise the retirement age. These issues are placed beyond the scope of this paper but should be born in mind in interpreting the results. Third, in this paper, whilst the calculations allow for relatively higher consumption levels for old people, reflecting their higher demands for health expenditures, we do not explicitly address how this consumption is actually distributed to older people. In practice a considerable part of this distribution operates through the tax, transfer and expenditure programs of government. As a result an ageing population has important implications for government outlays and taxation level. These implications may have efficiency effects that would feed back to the consumption possibilities. They may also meet with redistributional resistance, or tax revolt. We place these efficiency and distributional effects beyond the scope of this paper. However, using the projections of government outlays based on similar population projections in Guest and McDonald (2000) we comment on the distribution issue in the conclusion.

The paper is structured as follows. In section II the past demographic trends in Australia, which are the basis of the prospective ageing of the Australian population structure,  

7 Guest and McDonald (1999) investigate the impact on the support ratio and optimal saving of increasing the retirement age. It is found that the effect on optimal national saving of increasing the retirement age is negative but small, in the order of 0.5 to one percent of GDP for fairly large increases in the retirement age.
are briefly described. In section III the model of optimal national saving in a small open economy used in the simulations is described. Section IV describes how we allow for the effects of ageing on employment, labour productivity and consumption demands. Section V presents the simulations of optimal national saving and future living standards. Section VI concludes the paper.

II THE DEMOGRAPHIC PROFILE

The projections of optimal saving and living standards made in this paper are based on projections by age group and gender of the Australian population over the period 1999 to 2199. The base case population projection assumes that over the entire projection period the total fertility rate will be 1.75 and the rate of net immigration will be 0.54 percent of the population. These two assumed rates would continue the current actual rates for Australia. For mortality, it is assumed, following ABS(1998), that life expectancy at birth increases by 0.4 years every 5 years, a continuation of past trends. These assumptions imply that the population of Australia grows from 19 million in 1999 to 28.5 million in 2050.

Chart 1 shows the proportions of young and old people in the aggregate population implied by the base case population projection and the past history of these ratios back to 1950. The proportion of old people in the population was fairly constant up to the early 1970’s and then trends up. This upward trend is expected to be greater between 2010 and 2030. The proportion of young people in the population has been declining since its peak in 1961. This decline is expected to continue. The net effect of these patterns is shown by the total ratio of the two dependent groups. Currently that ratio is close to its lowest value for the period as a whole. It is expected to rise significantly after 2010, driven by the prospective ageing of the demographic structure, as shown by the increased proportion of old people. However this rise, which continues to 2050, will not yield unprecedented levels of dependency, at least as measured in the chart by numbers of people. In 2050 the total ratio of the two dependent groups is roughly equal to its value at the earlier peak, in 1961.

The prospective ageing of Australia’s demographic structure shown in Chart 1 is driven by the patterns of its total fertility rate, expectation of life and net immigration. For the past, these patterns are shown in the three panels in Chart 2. The total fertility rate, shown for the period 1936 to 1997, shows a substantial upward trend up to 1961 and then a rapid decline to the mid-1970’s, from which time the rate is fairly constant with some evidence of a slight decrease. The bulge in fertility is the major source of the prospective increase over the next 30
years in the proportion of dependents in the population.

The expectation of life at age 65, shown in Chart 2 for Australia for the period 1886 to 1995, shows a slight upward trend up to the mid 1960’s and then a somewhat steeper upward trend from then on. The pronounced kink in the trend in the mid-1960’s is greater for males than females.

Net immigration into Australia as a percent of population for the period 1949-50 to 1998-99 is shown in the third panel of Chart 2. The 10 year moving average shows that the rate was about 0.75 percent in the 1960’s and dropped in the mid 1970’s to around 0.54 percent, at which rate it has roughly remained.

It can be seen from these historical trends why the assumptions for fertility and immigration underlying our base case population projection can be described as a continuation of the current situation. The assumed total fertility rate and the rate of net immigration continue the rates of the last few years. However there are reasons for considering the possibility that in the future there may be decreases in both these rates. This is taken up later in this paper.

III  THE MODEL OF THE SOCIAlayanLY OPTIMAL LEVEL OF NATIONAL SAVING AND LIVING STANDARDS

Our simulations of the optimal levels of saving and consumption are based on the standard open economy model with many periods, see for example Obstfeld and Rogoff (1996, pp.60-2). However that model is modified in several ways to facilitate simulation of actual economies. These modifications include a positive rate of technical progress, a putty-clay vintage production function, a reference level of consumption in the representative consumer’s utility function and a changing demographic structure. By incorporating the last two features, the model in this paper extends the earlier version developed in Guest and McDonald (1998a). In this section we will describe the key features of the model. The equations of the model are in Appendix A. The choice of parameter values is described in Appendix B.

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8 In the Obstfeld and Rogoff section cited in the text, some of output, called government spending, is assumed not to add to consumption, investment or the accumulation of overseas assets. Our model does not include such a category of expenditure.
The model is one of a small open economy, that is an economy that can borrow or lend at an exogenously determined world rate of interest. For this economy, a social planner is assumed to maximise a social welfare function given by

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V = \sum_{j=1}^{h} \left[ N_j \left( \frac{C_j}{P_j} - \chi \frac{C_{j-1}}{P_{j-1}} \right)^{1-\beta} \frac{(1+\rho)^{1-\beta}}{(1-\beta)} \right] + N_h \omega \left( \frac{W_h}{N_h} \right)^{1-\beta} \frac{(1+\rho)^{1-\beta}}{(1-\beta)}
\]

with, for concavity, $\beta > 0$. In this function, social welfare is the sum of the utility levels generated from consumption running up to $h$ periods in the future and of the level of wealth at the end of the $h$ periods. To allow for the fact that consumption demands vary by age, which is important in assessing the best way to plan for an ageing population structure, the per capita measure of consumption in the social welfare function is $C/P$, that is consumption per consumption unit, where $C$ is aggregate consumption and $P$ is the population measured in consumption units, $P$. Consumption units weight people of various ages by their consumption demands. The calculation of these relative consumption weights is described below in section IV. As will be seen, the average person aged 75 years or more consumes 19 percent more than the average individual aged 25 to 39 years. This higher rate of consumption for very old people is driven by the higher expenditures on health. So, holding $C$ and $N$, which is aggregate population in natural units, constant, an increase in the proportion of very old people in the population will increase $P$ and thereby lower consumption per consumption unit. Wealth, defined as the sum of the domestic capital stock and overseas assets, is allocated at the end of the projection period equally to people, measured in natural units.

The utility function includes a reference level of consumption, specified as the level of consumption per consumption unit in the previous period. Thus the utility function allows for habit formation in consumption. In our simulations, the weight on lagged consumption, $\chi$, is set at one. Effectively the social planner is concerned with the growth of consumption. Including the reference consumption level into the social welfare function in this way implies that consumption has to have a positive rate of growth through the projection period.\(^9\) Furthermore, for the first period of the optimal plan we set the reference level of consumption

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\(^9\) Technically, if consumption per person in one period is less than consumption per person in the previous period then the social welfare function implies that the marginal utility of consumption per person is negative. This nonsense result has to be avoided. If, as is not the case for any of the simulations in this paper, the constraint on an economy ruled out the possibility of positive consumption growth then our specification of the social welfare function should not be used without modification. One modification to deal with such a situation would be to set the exogenous reference consumption per person for the first period at a sufficiently low level to allow a solution with positive growth in consumption.
equal to actual consumption in the previous year for the country under simulation. Hence in the optimal solution consumers do not suffer a reduction in their living standards from the level in the year preceding the plan.

The rate of time preference, $\rho$, is assumed constant over the planning horizon. It is set at the level that causes the ratio of consumption to GDP to tend towards a constant value in a steady state. The weight on terminal wealth, $\omega$, is set to protect the level of wealth, so that wealth at the end of the projection period has the same ratio to consumption as at the beginning.

On the production side, the economy produces one type of output. The vintage production function is putty-clay.\(^{10}\) The capital-labour ratio of new capital is chosen from a range of possibilities described by a Cobb Douglas production function. (This is the ‘putty’ aspect of capital.) Once installed capital has a fixed capital-labour ratio and depreciates physically at a fixed exogenously determined rate. (This is the ‘clay’ aspect of capital.) The age at which capital is scrapped is determined optimally. The capital is operated by an exogenously determined number of “representative” employees who reflect the relative labour productivities of the age/gender groups, as discussed below in Section III. This age distribution is another important factor to allow for in analysing an ageing population.

Finally, domestic expenditure, foreign borrowing/lending and output (GDP) are related by an international budget constraint.

In broad outline in the model consumption and saving behave in the following way. Fluctuations in consumption and saving are independent of each other. This is the Fisher separation result that arises in many models of the small open economy because the world rate of interest is exogenous. Consumption smoothing generated by maximising the social welfare function causes consumption per person\(^{11}\) to grow at a fairly constant rate. However the growth of output per person varies with changes in the demographic structure. Thus the optimal proportion of GDP saved is influenced by the prospective pattern of the ageing structure of the population. Broadly speaking, when the proportion of old people is increasing consumption per person grows more quickly than output per person, implying a decreasing rate of saving. Furthermore, when an ageing situation just described is anticipated to happen in

\(^{10}\) A vintage production function captures the influence of employment growth on the speed of introduction of new technology. Variable employment growth is a feature of demographic change.

\(^{11}\) Strictly speaking, consumption per consumption unit.
the near future the optimal response is to set the saving ratio at a high level. It will be seen below that the output from the simulations are in accord with these patterns.

IV ALLOWING FOR THE EFFECTS OF AGEING ON EMPLOYMENT, LABOUR PRODUCTIVITY AND CONSUMPTION

From the population projections we make projections of aggregate employment in efficiency units for the projection period. To make these projections we recognise that employment/population ratios and labour productivities vary by age and gender groups. Our method, following Cutler et al (1990), is to multiply the projected population levels by age and gender groups by the 1999 values of employment/population ratios and relative labour productivity by age and gender group. The resulting efficiency unit employment levels are summed over all age and gender groups to yield the aggregate employment level for the projected year. Employment/population ratios were calculated by dividing hours worked by employed persons divided by population by age and gender groups from data for 1999. The employment population ratios are reported in Table 1. The labour productivity weights were calculated on the assumption that the age distribution of earnings reflects the age distribution of labour productivity. The earnings distribution was measured by the distribution of mean weekly earnings of full time employees in their main job, ABS. Catalogue 6310.0 "Weekly Earnings of Employees". For years beyond 1999 we used the 1999 weights. The weights for each gender group are normalised on the mean wage that group. This boosts the female weights on average and may be thought of as implicitly assuming that all of the male/female differential is due to discrimination, rather than differences in productivity. The productivity weights are shown in Table 2.

Demands for both private and public consumption vary across age groups. For example, young people consume less private consumption but more education. Older people consume more health services. To allow for these relative demands for consumption, in our model social welfare is a function of consumption per consumption unit. Consumption units reflect the relative demands for consumption of different age groups. (We assumed no differences by gender). Thus an older person, who, as it will be seen, has a higher

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12 Most of the data used in this paper is in Australian financial years. When referring to that data we denote financial year x/x+1 as year x+1.
13 The hours worked data was supplied by the ABS from the Labour Force Survey.
consumption demand than other people will be represented by more than one consumption unit. Using population figures measured in consumption units gives an indication of how the changing age structure of the population over the period of projection will influence the social welfare created by aggregate consumption over this period. To construct the population figures measured in consumption units we calculated levels of private consumption and levels of government-provided consumption for various age groups. For private consumption levels we used the household expenditure survey for 1993-94 (ABS Cat No. 6530.0). We divided the population into three groups, young (0 to 19 years), working-age adult (20 to 64 years old) and old adult (65 years and older). In order to allocate total household consumption between young people and working age adults, we assumed, following the equivalence scale approach in Bradbury and Saunders (1990) that a young person consumes half the private consumption of a working-age adult. With this weighting, consumption per working-age adult per week was $268.69, $261.19 and $286.20 in households headed by persons of age 25-34 years, 35-44 years and 45-54 years respectively. On this basis we set private consumption per working-age adult at $272.03, the average across the three age groups. For old adults the HES data suggests that households headed by old adults are more or less entirely composed of old people. Dividing total consumption of households headed by old adults by the number of old adults gives a consumption per week per old adult of $202.61. We then adjusted these private consumption levels to 1996-97 values by multiplying by the ratio of total final consumption in 1996-97 to 1993-94. From these calculations the weights for private consumption on young, working-age adults and old adults are 0.5, 1 and 0.75. For government provided consumption we used figures from a detailed survey using 1988 data on Commonwealth and state social expenditure per person by age group for education and health by the Commonwealth Department of Community Services and Health (CDC) (1990), adjusting those figures to 1996-97 values. These figures are based on a division of the population into

14 Schultz and Borowski (1991), using the Consumer Expenditure Survey for the US, calculated consumption weights of 0.41, 1 and 0.85 for young, young adult and old adult respectively. Their method is broadly similar to ours, but somewhat more sophisticated in separating private consumption by the young from private consumption by young adults by estimating a consumption function across households. Cutler et al (1990) assume consumption weights for private non-medical consumption of 0.5, 1 and 1 for young, young adult and old adult respectively.

15 We adjusted the 1987-88 CDC estimates of government social outlays per head to the base year of our projections of 1996-97 values by a factor based on the ratio of total government social outlays for 1996-97 relative to 1987-88. In the adjustment the relativities between age groups was preserved. To do this we proceeded as follows. According to ABS Government Finance Statistics Cat no. 5512.0, total government social outlays grew from 18.1% of GDP in 1987-88 to 20.65% of GDP in 1996-97. To allow for the possibility that some of this increase was due to an increase in outlays per person and to allow for the increase in the price level, we adjusted the CDC outlays per person figures by an adjustment factor. For each of the three components of total social outlays, that is social security and welfare, education and health, the adjustment factor was the ratio of the total outlays for that component in 1996-97 to the sum of 1987-88 outlays per person by age for that component times the number of people in each age group in 1997. The three adjustment factors were 1.94 (social security and
nine age groups. Combining the private consumption expenditures with the government consumption expenditures yielded the consumption weights for the nine age groups given in Table 3. Multiplying the consumption weights in Table 3 by the population in each age group and summing yields the aggregate population measured in consumption units.

Chart 3 plots various support ratios, the ratio of employment to population, for Australia for the period 1971 to 2050. Up to 1999, actual employment is used. Beyond 1999 the employment figures are based on the population projections as described above. The base(unweighted) support ratio assumes that labour productivity and consumption demands are the same across age and gender groups, for Australia for the period 1971 to 2050. It can be seen from Chart 3 that the unweighted support ratio increases from 1971 to a peak in 2001 and then decreases for the rest of the period to 2050.\textsuperscript{16}

The base case support ratio shown in Chart 3 incorporates the weighting across age and gender groups as described above. It can be seen that this adjustment makes little difference to pre-1999 data but following 1999 the weighted support ratio decreases relative to the unweighted support ratio. The weight-adjustment to the support ratio reflects two effects. The productivity weighting increases the support ratio in an ageing workforce because older workers tend to have higher marginal products. However, the weighting of consumption demands reduces the support ratio in an ageing population because old people over 75 demand significantly more health services. This later effect dominates from 1999 onwards. Note also that in 2050, by which time the ageing effect has substantially occurred, the weighted support ratio is only a little below its 1971 level. (The other support ratios in Chart 3 are discussed below.)

The projections of the support ratios in Chart 3 and thus the employment and population series used in our simulations of optimal national saving assume that the consumption weights, productivity weights and employment population ratios will not change over time. These are strong assumptions. They ignore the possibility that in as far as the decline in the support ratio places a pressure on resources there will be some adjustments to expenditure patterns, productivity or employment-population ratios. A pressure on resources, and the improved health of old people, can be expected to increase the employment-population ratios for old people. On the other hand, trends in health expenditures in the US suggest that

\textsuperscript{16} For the years before 1999 the ratio shows fluctuations. These reflect cyclical fluctuations in employment.
our measures of the support ratio in Chart 3 underestimate the prospective burden of the aged. It is sometimes argued that health expenditures may increase faster than labour productivity if recent trends continue into the future. For example Hurd (1997) points out that for the US from 1970 to 1990 health expenditures per person (private plus government) have been increasing by 4 to 5 percent per year. In as far as health expenditures on the aged in Australia are expected to increase faster than the growth of labour productivity, the support ratios in Chart 3 underestimate the prospective burden of the aged. Over the projection period the productivity weights would not be expected to stay constant, because of, among other things, changes in human capital and demand and supply for particular groups. However because of the difficulty in projecting such changes we assume the productivity weights are unchanged over the projection period. It will be seen below that the weights in fact have a small effect on the rate of optimal national saving.

V SIMULATIONS OF OPTIMAL NATIONAL SAVING AND FUTURE LIVING STANDARDS

In this section we report the simulations of optimal national saving and future living standards for Australia using the model described above. The parameter values are in the appendix. Although all simulations reported in this section assume a 170 year horizon, we focus on the results for the period up to 2050. We ignore the post 2050 period because, being so far in the future, it is of less interest. We also emphasise that the simulations protect the initial level of wealth in that the terminal condition for all simulations requires a wealth-consumption ratio equal to the initial wealth-consumption ratio. Thus the simulations do not run down wealth over the 170 year horizon.

The base case

The rates of saving, investment and the current account balance for the base case are reported in Chart 4. For optimal national saving the simulation with this assumption from 1999 shows a hump shape over the projection period. There are five features of this hump shape.

1. In 1999 the rate of optimal national saving is equal to the actual rate for Australia in 1999.
2. The hump peaks in year 2010.
3. The size of the hump relative to the initial year is equal to 3.2 percent of GDP,
reflecting an increase in optimal national saving from 19.0 percent of GDP in 1999 to 22.2 percent of GDP in 2010.

4. The decrease in optimal national saving after the peak from 2010 to 2050 is 5.3 percent of GDP.

5. In year 2030 the optimal rate of national saving is the same as in 1999.

The initial 30 years of the projection period, from 1999 to 2030, are years of high saving, relative to the rest of the projection period. As explained above, this high optimal rate of national saving reflects in part the anticipation of much lower support ratios in the future. Around 2030 the years of a high support ratio will be over. The support ratio will have fallen to a level similar to those of the 1970’s and 1980’s. Subsequently the support ratio will be lower. The high rate of national saving in the period 1999 to 2030 generates an increase in wealth from which consumption in the following years of relatively low support ratios can be financed. This is of course how increased saving can help to reduce the pressure on resources imposed by the ageing population.

The simulation of the base case also shows, see Chart 4, that the optimal disposition of the increased saving rate is to reduce the amount of borrowing from overseas rather than to increase the domestic capital stock.\textsuperscript{17} This can be seen by the increasing optimal current account balance as a share of GDP (CAB/Y) shown in Chart 4. The optimal plan for Australia is to decrease the current account deficit by 5.3 percentage points of GDP from 1999 to 2010. The optimal level of investment as a share of GDP (I/Y) decreases from 24.5 percent of GDP in 1999 (slightly above its actual level of 23.8 per cent of GDP in 1999) to 22.2 percent in 2010. This decrease is driven by the slowing of employment growth in those years. The decrease in the optimal national saving rate after 2010 is associated with a decreasing optimal current account balance. The current account deficit increases by 5.4 percentage points of GDP from 2010 to 2051.

From the simulations the future pattern of living standards implied by Australia following the optimal path for national saving can be recovered. As pointed out in the introduction, these are the best possible outcomes for living standards. We identify living

\textsuperscript{17} A lower world rate of interest will change the optimal disposition of national saving by increasing investment and overseas borrowing. For example simulations with the model show that a world rate of interest of 3 per cent will increase the optimal level of investment to 38.7 per cent of GDP and increase the optimal current account deficit to 17.8 per cent of GDP. The subsequent pattern is the same as for the base case reported in the text.(Of course by historical standards a world rate of interest of 3 per cent is an extraordinarily low level. It gives the direction of the effects.)
standards with consumption per consumption unit (C/P). By using consumption units we allow for a relatively higher level of consumption to be allocated to older people, by the amounts given in Table 3 (and lower levels to children). The levels of C/P for Australia for the period 1971 to 2050 are reported in Chart 5. For the sub-period 1971 to 1999 the graph shows the actual level of C/P; for the sub-period 1999 to 2050 the chart shows the projected level of C/P, based on the optimal saving outcome for the base case. It can be seen that living standards are projected to increase from 1999 to 2050. The average annual rate of growth of C/P over this period is 1.01 percent. This is 0.32 percentage point less than the average annual rate of growth of labour productivity over this period, this difference reflecting the impact of the ageing population structure. It is also less, and by a larger amount, than the growth of living standards up to 1999. From 1971 to 1999 C/P grew at an annual rate of 1.9 percent. The high growth of C/P in the past reflects partly the “younging” of the Australian population up to 1999, as shown by the increasing support ratio for the period 1971 to 1999 shown in Chart 3.

The most important conclusion to be drawn from Chart 5, in our view, is that living standards in Australia are projected to improve, even allowing for the ageing structure of the population. This is the case even with no immediate cut in living standards. Future generations will be better off. It is the assumed continuation of the rate of total factor productivity growth of one per cent per year, observed over the last century, that is the main driving force in the increasing standard of living. Indeed experimentation reveals that the rate of total factor productivity of 0.3 per cent per year would yield a constant living standard along the optimal path.

It is worth emphasising that along the optimal path living standards will improve even although the rate of time preference is assumed to be positive. Sometimes people argue that a positive rate of time preference discriminates against future generations. However with a positive rate of growth of total factor productivity and our procedure for choosing the rate of time preference this is not true.

The impact of ageing

It was pointed out above that the prospective ageing of the Australian population will drive a wedge between the rate of growth of labour productivity and the rate of growth of living standards of 0.32 per cent per year for the period 1999 to 2051. This is a measure of the ageing burden. Another way to measure the ageing burden is to consider a simulation which assumes a homogenous population with respect to age. This simulation is reported in Charts 6.
and 7, labelled Base (HOM). The basis of this simulation is to take the aggregate population implied by the base case projection and to assume that the aggregate employment/population ratio remains at its 1999 level throughout the 170 year projection period. Furthermore labour productivity and consumption demands are assumed to be the same across age groups. Chart 6 shows that the optimal rate of national saving is fairly constant for the Base (HOM) case, at around 20 percent of GDP. Chart 7 shows that the implied rate of growth of living standards would be greater: by 2030 C/P would be 8.8 percent higher and by 2050 11.2 percent higher if the age structure of the population remained unchanged.

**The case for using the reference level of consumption**

The use of a reference consumption benchmark in the social welfare function is an important part of our method. Because it may appear to some economists somewhat unconventional, in that the traditional approach is to base utility on levels of consumption, not the change in levels of consumption, in this section we make the case for including reference consumption. To do this we compare our results with those obtained from dropping reference consumption.

To drop reference consumption, we ran the simulation of optimal national saving for the base case with $\chi$ in the social welfare function set equal to zero. The resulting patterns of optimal national saving and living standards are shown in Charts 6 and 7, labelled Base (NRC). With no reference consumption, the optimal pattern of national saving, see Chart 6, is an immediate increase in 1999 to 25.4 percent of GDP, an increase of 6.6 percent of GDP over the actual level in 1999. To do this would require an immediate cut in living standards. This is shown in Chart 7. C/P would be 8.8 percent below the optimal outcome for the base case with reference consumption. The reward for this cut in living standards is higher living standards in the future. It can be seen from Chart 7 that C/P for the Base (NRC) case grows at a faster rate than for the base case and overtakes the base case around 2020. Thereafter C/P would be higher.

What then is the case for choosing the social welfare function with reference consumption? Choosing whether to include reference consumption is choosing between 2 specifications of the social welfare function. In the context of our model, this choice should be made on the basis of comparing the paths of C/P. Which path of C/P, the base case or the Base (NRC), is to be preferred? If it is the one implied by the base case then a social welfare

---

18 The support ratio for the homogenous population is by definition unchanging over time.
function with reference consumption is preferred over a social welfare function with no reference consumption. The case against the social welfare function with no reference consumption is that it implies an immediate and substantial cut in C/P, something that it is reasonable to judge that most people would strongly dislike. (That people strongly dislike cuts in consumption is of course the basis for including reference consumption in the social welfare function.) Furthermore, as emphasised above, the base case simulation shows that for Australians even with no cut in current living standards there can be consistent growth in future living standards. This strengthens the argument that people would prefer the social welfare function with reference consumption. The gain from choosing NRC as the basis for optimisation is a long way off, 20 years hence.

Remembering that the optimal path of C/P with no reference consumption does not overtake the optimal path of C/P with reference consumption until 2020, one can infer some intergenerational conflicts in choosing the social welfare function. Older people would tend to prefer the social welfare function with reference consumption because they will not be around after 2020 to enjoy the gains from current belt tightening (ignoring any concerns they may have for their descendents). On the other hand those to be born in the future, especially after 2020, do better from the optimal path without reference consumption. However, note that existing generations can assuage any guilt they may feel from enjoying the benefits from choosing to optimise with reference consumption by the expectation that unborn people will still be better off than they are.

The impact of reduced immigration

In recent years there has been in Australia a vigorous expression by some parts of the Australian community of a desire for lower immigration. Because the rate of immigration influences the age structure of the population, it may have an effect on optimal saving and the implied future living standards. To gain some idea of the quantitative size of this impact we compared the base case with one other population projection. In this other population projection, labelled ZIMM, the rate of net immigration is assumed to be zero per year for 1999 to 2199. This implies a population of Australia of 19.5 million in 2050. Lower immigration intensifies the ageing problem. This can be seen from the support ratio labelled ZIMM in Chart 3. This support ratio falls by more over the next 50 years than the support ratio for the base case. The implication for the socially optimal levels of national saving of the ZIMM population projection is reported in Chart 6. The hump shape pattern is repeated. The hump peaks in the year 2010 at the same level as the base case. However, thereafter, with zero
immigration the optimal pattern for saving is less than for the base case. Not surprisingly, the implied path for living standards is lower with zero immigration, as shown by Chart 7. By 2030 C/P is 6.4 percent lower and by 2050 C/P is 11.2 percent lower. Thus a reduced rate of immigration tends to reduce the best possible outcome for future living standards.

The reason for the negative effect on living standards of reduced immigration is through its impact on the age distribution of the population. The base case population projection follows the ABS in basing the age composition of immigrants on the average structure between 1994-95 and 1996-97, see ABS (1998, p.35). In Chart 8 this age structure is compared with the age structure of the Australian population in 1999. Clearly immigrants are heavily concentrated in the 15 to 35 year age group with relatively few in the 50 years and over age group. This age structure of immigrants tends to reduce somewhat the ageing of the population.

The impact of reduced fertility

Several commentators, for example Peter McDonald (1999, p.52) and Craig Thornburn (1999) have argued that in the near future the total fertility rate in Australia will fall below the 1.75 rate assumed in the base case. The reason for this view is partly based on overseas experience. As McDonald (1999, p.52) says, “most industrialised countries have fertility levels at present that are lower than the Australian level”. Most notable are Germany and Italy, where the fertility rate had declined to 1.3 in the period 1990-1995 (OECD (1996, p.104). It is generally believed that low fertility is bad for living standards. For one example from the media, Maureen Freely recently wrote in the Guardian “Our birth rate is sinking…and you get a serious problem in the long term. In the decades to come, we will have an ever smaller pool of working age adults and they will have to support an ever larger pool of pensioners” (Freely (2000)).

There are two caveats to the arguments of the previous paragraph. Firstly, the decrease in fertility may not occur or may be only a temporary phenomenon. As a contrast to the experience of Germany and Italy noted above, several comparable OECD countries have higher rates of fertility than Australia. In the 1990-1995 period the US had a fertility rate of 2.1. Sweden and New Zealand also had fertility rates of 2.1. For Canada the rate was the same as Australia. The UK was only slightly lower than Australia, at 1.8. Thus Germany and Italy are not the only alternatives. Furthermore it has been argued that the recent decrease in fertility
may be reasonably expected to stop and even to reverse in the future, see Bongaarts (1999). This argument is based on the proposition that the reduction in the fertility rate currently observed reflects a transition to having children at later ages and so is temporary. The plausibility of this proposition is suggested by the evidence from surveys that most couples plan to have about two children over their lifetimes. Secondly, even if fertility falls, living standards may not be much affected. The only rigorous attempt in the literature of which we are aware to quantify the impact of reduced fertility on living standards finds that “(A) stable population with a growth rate …1 percentage point … below the optimum…would have a level of consumption only 2.1 percent below that of an optimally growing population” (Weil (1999, p.252)). Furthermore Weil shows that in the transition to a lower population growth rate there is “an initial period of lower-than-steady-state dependency, and thus unusually high consumption” (Weil (1999, p.253)).

To investigate for Australia the impact of reduced fertility on living standards we consider two population projections with reduced fertility. One, labelled PMcD, is based on a fertility projection that Peter McDonald considers plausible for Australia, that is the total fertility rate falls from 1.75 in 1999 to 1.65 in 2004 at which level it remains for the next 200 years. The second, labelled TFR1, assumes that the total fertility rate drops immediately to 1 in 1999 at which rate it remains for the next 200 years. TFR1 might be considered below the bounds of probability by most demographers. However a perspective on this assumption is provided by Golini(1998). Golini argues that a total fertility rate of 0.8 is the lowest practical outcome for present day populations of large size. His argument is that empirical evidence suggests that the highest proportion of women to be childless throughout life is 20 percent and that for the remaining 80 percent a choice of one child per female lifetime is the lowest number. Golini points out that total fertility rates of 0.8 have been observed, in East Germany during 1992-95 and in the province of Ferrara, Italy in 1994. In the light of Golini, our TFR1 may be thought of as the lowest plausible extreme. It is an unlikely outcome but useful for illustrating the implications of low fertility.

Apart from the total fertility rates, PMcD and TFR1 are based on the same underlying assumptions as the base case, in particular the rate of immigration is 0.54 percent of population. The implied populations for Australia for 2050 are 27.4 and 20.5 million
respectively.

The socially optimal levels of national saving implied by the PMcD and TFR1 population projections are reported in Chart 6. The hump shape pattern for optimal national saving is repeated in both cases, but with significant differences compared with the base case. For PMcD the peak of the hump occurs later in 2018 and subsequently the rate of decrease in optimal national saving is smaller. For TFR1 the hump also peaks later, in the year 2022. However the rate of national saving is much higher, being 29.5 percent of GDP at the peak. This is an increase of 10.7 percent of GDP over the 1999 rate. The implied path for living standards, as shown by Chart 7, is virtually the same for PMcD and is slightly higher for TFR1. Thus a reduced rate of fertility does not reduce the best possible outcome for future living standards.

The results of the low fertility simulations is consistent with Weil’s (1999) point that there is a gain of reduced dependency in the short term from a reduction in the total fertility rate. This is especially apparent for the TFR1 case. The support ratio for the TFR1 case shown in Chart 3 is increasing from 1999 to 2014. The reduction in the number of children causes this. Our simulations show that a large increase in national saving makes best use of this immediate gain. And this increase in the saving ratio does not reduce living standards relative to the base case. From this we conclude that TFR1 shows very clearly that reduced fertility is not a threat to living standards if the rate of saving is increased.

VI CONCLUSION

Australia faces a significant ageing of its population in the next 30 years. On the basis of calculations of the levels of optimal national saving and the implied living standards for Australia for the period from 1999 to 2050, this paper suggests the best path for nation saving to meet this challenge. That path is an increase in national saving of 3.2 percent of GDP by 2010 followed by a steady decrease to 16.9 per cent of GDP by 2050. The increase to 2010 should be devoted to reducing the current account deficit rather than increasing the level of

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19 In as far as the actual rate of saving for Australia is influenced by the optimality considerations, our results can be interpreted as a forecast of the pattern of actual saving and living standards in Australia in the future.
investment in the domestic capital stock. For living standards, following this path would lead to a growth in consumption per person of about one percent per year over the next 50 years. This would not cause any running down of the wealth of Australians.  

The program outlined in the previous paragraph is, on the basis of our approach, model and value judgements, the best that Australia can do. The value judgements are incorporated in the social welfare function we chose. In this function, social welfare is based on consumption relative to a reference level. We specified the reference level as the previous period’s consumption. Thus social welfare depends effectively on the growth of consumption, not the level of consumption.

The major outcome of our simulations is the paths of consumption per consumption unit, as shown in chart 7. Consumption units allow for the different demands for consumption by age. Of particular importance is the relatively high level of consumption per person of old people due to high expenditures on health. We use the summary statistic of the average annual rate of growth of consumption per consumption unit for 1999 to 2050, presented in Table 4, to summarise our conclusions. For the base case, which roughly continues the current demographic picture of Australia, C/P grows at 1.01 per cent per year from 1999 to 2050. It is the growth of total factor productivity that drives the future growth of C/P. That is to say, productivity growth dominates the ageing effect.

Optimising without reference consumption in the social welfare function would yield a path with a higher rate of growth of C/P, 1.36 per cent per year, but at the cost of an immediate cut in C/P of 8.8 per cent. C/P would not catch up with the base case outcome until 2020. We judged this outcome to be inferior to the outcome based on reference consumption, that is the base case.

The optimal rate of growth of consumption per consumption unit in the base case is half the rate of growth of C/P enjoyed over the 30 years preceding the start of our simulations, from 1971 to 1999. (The high growth of C/P in the past was aided by the younging of the population.) Thus this optimal plan requires Australians to adjust to a lower rate of

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20 Some people have questioned our assumption of a fixed world rate of interest over the projection period, on the basis that the ageing of the world population will cause high saving which will lower the world rate of interest. However, any tendency for saving to increase in future, and note that our projections suggest that optimal saving decreases as the ageing process takes effect, that is after 2010 in Australia and probably earlier in other OECD countries, may be accompanied by the increased investment opportunities from development in the non-OECD world. The net effect on the world rate of interest is ambiguous. Further investigation of this issue is warranted.
consumption growth than they have recently experienced. Some may be tempted to argue that
the case with no reference consumption is a preferable outcome because it implies a smaller
shortfall from the past rate of growth of consumption. But of course this comes at a cost of an
immediate and large cut in consumption! It is not really a way out. Furthermore, it can be
argued that our base case treats future generations generously in that it implies that they will
be better off than people are today.21

One measure of the impact of ageing on living standards is provided by our simulation
of a homogenous population, that is one in which the effects on relative consumption
demands, employment participation and labour productivity of changes in the age distribution
of the population are ignored. For this simulation, labelled HOM, the average rate of growth of
C/P is 1.27 per cent per year. Thus ageing reduces the best possible outcome for C/P growth
by 26 per cent.

Reduced immigration was shown to reduce consumption growth. Comparing the case
of zero immigration, labelled ZIMM, with our base case of immigration equal to 0.53 per cent
of population reduces the rate of growth of C/P to 0.78 per cent per year, a reduction of 23 per
cent.

Reduced fertility does not reduce living standards. Allowing the total fertility rate to
fall to 1.65 by 2004, an outcome deemed likely by commentators such as Peter McDonald and
Craig Thornburn, has virtually no effect on C/P, see the column labelled PMcD. More
spectacularly, an immediate reduction in the TFR to 1, an extremely low level, actually leads
to a slight increase in the rate of growth of C/P, to 1.05 per cent, as shown in the column
labelled TFR1.

Our variations in demographic assumptions focus on downside risks, although one,
reduced fertility, turned out to be an upside risk. Other of our assumptions are conservative
and understate the fruit of the future. Especially, our assumption that employment/population
ratios are fixed in the future at 1999 levels probably understates the future level of
employment. The increasing dependency ratio in the future will probably cause the
employment/population ratios of many groups in the population to increase, in particular the
old through later retirement, the young through reduced participation in education and reduced

21 Because we calculate C/P we can make more definite conclusions than could Cutler et al (1990). Compare
especially the ambiguous tone of the discussion.
unemployment, and females through increased labour force participation. These increases in employment/population ratios will increase GDP in the future and thereby increase future living standards.

From these conclusions it would appear that for Australia ageing will not lead to lower living standards. However some care must be taken in drawing this conclusion. First, to achieve the C/P paths described above requires an increase in national saving of 3.2 per cent of GDP up to 2010 and a reduction in the current account deficit of 5.3 per cent of GDP. These are not easy targets to achieve. However should we fall short of them, the effect on future living standards is probably very small. Second, the paper does not address the distributional issue of how the consumption will be delivered to the old people. In practice the government through tax, transfer and expenditure systems plays a major role. In another paper, using assumptions consistent with those in this paper, we find that government outlays will rise by about 5 per cent of GDP because of the ageing population. We are inclined to the conclusion that in as far as ageing is a problem it is a problem of government outlays, not a problem of too little consumption to go round.
APPENDIX A – THE MODEL

In this appendix the model used to calculate the optimal levels of national saving in the text is described. The model is based on the following maximisation problem.

Maximise \[ \Gamma = V(C_1, C_2, \ldots, C_h, W_h) \]

\[
\sum_{j=1}^{h} \lambda_j [Y_j + B_j - I_j - C_j - D_0 (1-m)^{j-1} (m + r_0) - \sum_{k=1}^{j-1} B_k (1-m)^{j-k-1} (m + r)] 
+ \sum_{j=2}^{h} \psi_j \{ [\sum_{k=1}^{T_{j-1}} (1-\delta)^{k-1} A_{j-k} F(I_{j-k}, I_{j+1-k})] - Y_j \} 
+ \xi_f \sum_{j=2}^{h} [L_j - \sum_{k=1}^{T_{j-1}} (1-\delta)^{k-1} I_{j+1-k}] 
+ \phi [\sum_{k=1}^{h-T_{j-1}} (1-\delta)^k I_{h-k}] - D_0 (1-m)^h - [\sum_{k=1}^{h} B_k (1-m)^{h-k}] - W_h \]

by choice of

\( (C_1, C_2, \ldots, C_h) \)
\( (I_1, I_2, \ldots, I_{h-1}) \)
\( (B_1, B_2, \ldots, B_h) \)
\( (Y_2, Y_3, \ldots, Y_h) \)
\( (l_2, l_3, \ldots, l_h) \)
\( (T_2, T_3, \ldots, T_h) \)
\( (W_h) \)

where

Subscript=time period
C=aggregate consumption
W=wealth=capital stock plus foreign assets
B=borrowing from foreigners
I=aggregate investment
Y = aggregate output
D = foreign debt
m = proportion of debt repaid in each period
r = world interest rate
δ = rate of depreciation
l = number of workers on a vintage of capital in its first period of use
T = age of oldest capital good in use
h = terminal period of the maximisation problem
λ_j, j = 1, ..., h, ψ_j, j = 2, ..., h, ξ_j, j = 2, ..., h, and ϕ are Lagrange multipliers.

T_j is the age of the oldest capital good in use at time j. T_j^* is the (expected) age of scrapping of a capital good installed (built) at time j. The production function F(.) has constant returns to scale. The summation signs go from low, after the equal sign, to high. They only apply if high is an integer equal to or greater than low.

Exogenous variables:

(1_{2-T_2}, ..., 1_I)

(1_{3-T_2}, ..., 1_I)

(r)

(Y_1)

(D_0)

The specific form of the social welfare function is:

\[ V = \sum_{j=1}^{h} \left( N_j \left( \frac{C_j}{P_j} - \frac{C_{j-1}}{P_{j-1}} \right) \right)^{1-\beta} \left( \frac{1+p}{1-\beta} \right) + N_h \omega \left( \frac{W_h}{N_h} \right)^{1-\beta} \left( 1+p \right)^{1-h} \left( \frac{1}{1-\beta} \right) \]

The higher consumption demands of older people are accounted for by distinguishing between N, the number of people in natural units, and P, the number of people in effective units which are found by summing the consumption-demands-weighted number of natural
units in each age group. The lagged value of consumption per consumption unit, $C_{j-1}/P_{j-1}$, is the reference level of consumption. $\chi$ is a weighting factor.

The production function is a vintage putty-clay form. At the time of the investment decision the technique of production can be chosen from a Cobb-Douglas function

$$A_{j-1}I_{j-1}^{\alpha}l_{j-1}^{1-\alpha}$$

where $l_j$ is labour employed on capital of vintage $j$. If the rate of interest and the rate of technical progress i.e. the rate of increase in $A$, are assumed constant then the optimal age of the oldest capital good in use is constant. This implies that the production function can be written

$$Y_j = \sum_{k=1}^{T} \left(1-\delta\right)^{k-1} A_{j-k} I_{j-k}^{\alpha} l_{j-k}^{1-\alpha} \right)$$

for $j=2,...,h$

With these functional forms the first order conditions imply

(A1) \( T = \frac{-(1-\alpha)\ln(1-\alpha)}{\ln(1+a)} \)

(A2) \( E_j = \frac{1}{\sum_{k=1}^{T} (1+r)^{-k} (1-\delta)^{k-1}} \)

for $j=1-T,...,h-T$

(A3) \( E_j = \frac{1-(1+r)^{j-h}(1-\delta)^{h-j}}{\sum_{k=1}^{h-1} (1+r)^{-k} (1-\delta)^{k-1}} \)

for $j=h-T+1,...,h-1$

(A4) \( k_j = \frac{I_j}{l_{j+1}} = \left[ \frac{\alpha A_j}{E_j} \right]^{1-\alpha} \)

for $j=1-T,...,h-1$
(A5) \[ I_j = \frac{I_{j-1}}{k_{j-1}} \quad \text{for } j=2,-T,\ldots,1 \]

(A6) \[ I_j = L_j - (1-\delta)L_{j-1} + (1-\delta)^T I_{j-T} \quad \text{for } j=2,-\ldots,h \]

(A7) \[ I_j = k_{j-1} I_{j+1} \quad \text{for } j=1,-h-1 \]

(A8) \[ Y_j = \sum_{k=1}^{T} \left[ (1-\delta)^{k-1} A_{j-k} I_{j-k} I_{j-(k-1)}^{1-\alpha} \right] \quad \text{for } j=2,-h \]

(A9) \[ c_j = \frac{C_j}{P_j} - \chi \frac{C_{j-1}}{P_{j-1}} \quad \text{for } j=1,-h \]

(A10) \[ \left( \frac{N_j}{P_j} \right) \left( \frac{c_h}{c_j} \right)^{\beta} + \chi \left( \frac{N_{j+1}}{P_h} \right) \left( \frac{c_h}{c_{j+1}} \right)^{\beta} (1+\rho)^{-1} = \frac{(1+r)^{h-j}}{(1+\rho)^{h-j}} \quad \text{for } j=1,-h-1 \]

(A11) \[ W_h = \omega^{1/\beta} \left( \frac{P_h}{N_h} \right)^{(1-\beta)/\beta} C_h \]

(A12) \[ Y_j = I_j + C_j + D_0 \left( 1-m \right)^{j-1} \left( m+r_0 \right) - B_j + \sum_{k=1}^{j-1} B_k \left( 1-m \right)^{j-k} \left( m+r_k \right) \quad \text{for } j=1,-h \]

(A13) \[ W_h = \left[ \sum_{k=h-T+1}^{h-1} (1-\delta)^k I_{h-k} \right] - D_0 \left( 1-m \right)^{h} - \left[ \sum_{k=h}^{h-1} B_k \left( 1-m \right)^{h-k} \right] \]

Exogenous variables
From these equations the calculations in the paper are made.
APPENDIX B – DATA AND CALIBRATION

The set of parameter values used for all the simulations is listed in Table B1. They were chosen as follows.

For the production function the values of \( \forall \), the elasticity of output with respect to capital, and \( * \), the rate of depreciation, are based on typical empirical estimates and are the same as those used by Barro and Sala-I-Martin (1995, p. 83). The value of \( a \) is calculated from fitting the production function with the assumed values of \( \alpha \) and \( \delta \) to Australian data over the period 1965 to 1999. The value of \( A \) in the first year of the projection period is set such that the implied value of GDP is equal to the actual value of GDP 1999. This procedure calibrates the production function to actual data for the Australian economy.

\( r \), the world rate of interest is the same as used by Barro and Sala-I-Martin (1995, p. 124). The value of \( m \), the proportion of debt to be repaid in each year, is set at 0.15 to approximate a 10 year loan.

For the social welfare function \( \exists \), the reciprocal of the elasticity of intertemporal substitution, is based on typical empirical estimates, in particular Powell (1974), who estimates \( \exists \) to lie between 1.6 and 2.8 using Australian data, and the more recent estimates in Skinner (1985). Barro and Sala-I-Martin (1995, p.72) assume \( \beta =3 \), which would not make any difference to our calculations. \( T \) is set to generate a terminal value of wealth to consumption equal to the exogenously given initial value of 2.6, using (A11) in Appendix A. The value of the rate of time preference, \( \Delta \), is chosen so that the asymptotic growth rates of consumption and output are equal. In a model with zero technical progress this is the familiar restriction that \( \Delta = r \). In the case of positive technical progress and the Cobb-Douglas production function the corresponding restriction is that

\[
\rho = \frac{(1+r)^{\beta}}{(1+a)^{1-\alpha}} - 1
\]  

(B1)
Although some regard setting the rate of time preference by condition (1) as unacceptable (because it appears to relate the value of one exogenous variable to the values of other exogenous variables), we argue that in the current context it is a reasonable procedure. As we show in Guest and McDonald (1998b), if the rate of time preference is determined endogenously by using Uzawa preferences, see Uzawa (1968), then it will tend to the value determined by equation (1). (For other discussion of using this condition see Blanchard and Fischer (1989) and Barro and Sala-I-Martin (1995)).

The planning horizon, \( h \), is chosen to be long enough so that the path of optimal national saving to output, \( S/Y \), for the period up to the year 2051 is sufficiently close to the path that would obtain for an infinite horizon. The criterion for "sufficiently close" is that a further extension of the horizon would change the value of \( S/Y \) in the year 2051 by less than a level of tolerance specified as 0.1 percentage points. The resulting value of \( h \) is 170 years.

**Table B1** Values of parameters and exogenous variables

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>( \forall ), the partial elasticity of output with respect to capital</td>
<td>0.3</td>
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<tr>
<td>( * ), the depreciation rate</td>
<td>0.05</td>
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<td>( a ), rate of technical progress</td>
<td>0.01</td>
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<tr>
<td>( r ), interest rate</td>
<td>0.06</td>
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<tr>
<td>( m ), the proportion of debt to be repaid in each year</td>
<td>0.15</td>
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<tr>
<td>( \exists ), the reciprocal of the elasticity of intertemporal substitution</td>
<td>2.00</td>
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<tr>
<td>( \omega ), the weight on terminal wealth</td>
<td>6.788</td>
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<tr>
<td>( \Delta ), the rate of time preference</td>
<td>0.03122</td>
</tr>
<tr>
<td>( h ), the planning horizon</td>
<td>170 years</td>
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### Tables

#### Table 1 Employment/population ratios

(hours per week per person)

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<tbody>
<tr>
<td>males</td>
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<td>26.6</td>
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<td>34.1</td>
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<td>35.9</td>
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<td>15.4</td>
<td>8.6</td>
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<td>females</td>
<td>8.7</td>
<td>20.9</td>
<td>19.5</td>
<td>19.5</td>
<td>18.4</td>
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<td>19.1</td>
<td>11.3</td>
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#### Table 2 Labour force productivity weights

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<td>males</td>
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<td>1.07</td>
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<tr>
<td>females</td>
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#### Table 3 Consumption weights

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<th>Age group</th>
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<th>16-24</th>
<th>25-39</th>
<th>40-49</th>
<th>50-59</th>
<th>60-64</th>
<th>65-69</th>
<th>70-74</th>
<th>75+</th>
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</thead>
<tbody>
<tr>
<td>consumption weight</td>
<td>0.68</td>
<td>0.89</td>
<td>1.00</td>
<td>0.98</td>
<td>1.00</td>
<td>1.05</td>
<td>0.87</td>
<td>0.95</td>
<td>1.19</td>
</tr>
</tbody>
</table>

#### Table 4 Average annual rate of growth of consumption per consumption unit, 1999 to 2050

<table>
<thead>
<tr>
<th>Case</th>
<th>Base</th>
<th>NRC</th>
<th>HOM</th>
<th>ZIMM</th>
<th>PMcD</th>
<th>TFR1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Growth of C/P</td>
<td>1.01</td>
<td>1.36</td>
<td>1.27</td>
<td>0.78</td>
<td>1.01</td>
<td>1.05</td>
</tr>
<tr>
<td>Ratio to growth in base case</td>
<td>1.00</td>
<td>1.35</td>
<td>1.26</td>
<td>0.77</td>
<td>1.00</td>
<td>1.04</td>
</tr>
</tbody>
</table>
References


Economic Planning and Advisory Council (1994) Australia’s Ageing Society, EPAC Background Paper No 37, AGPS, Canberra.


Chart 1 Population ratios for dependent groups, Australia, 1950 to 2050
Chart 2 Fertility, expectation of life and immigration, Australia

**Total fertility rate, Australia, 1936 to 1997**

**Expectation of life at age 65, Australia, 1886 to 1995**

**Natural increase and net immigration, Australia, 1949-50 to 1998-99**
Chart 5 Living standards, base case, Australia, 1971 to 2050
Chart 6 The Impact of Ageing, Reference Consumption, Immigration and Fertility on Optimal National Saving, Australia, 1999-2050

Chart 7 The Impact of Ageing, Reference Consumption, Immigration and Fertility on Consumption per Consumption Unit, Australia, 1999-2050
Chart 8 Age distribution of net immigration and 1999 population

% imm
% pop