A model of longevity, fertility and growth

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Abstract

Economic and demographic outcomes are determined jointly in a dynamic general equilibrium model of longevity, fertility and growth. Reproductive agents in overlapping generations mature safely through two periods of life and face an endogenous probability of surviving for a third period. Given this probability, each agent maximises her expected lifetime utility by choosing consumption and the number of children. Child-bearing is costly in the sense that time must be spent on child-rearing activities rather than on production or education. The model produces multiple development regimes which yield different predictions about life expectancy, fertility, timing of births and educational investment depending on initial conditions. These predictions accord strongly with the empirical evidence on demography and development.

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1. Introduction

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Greater life expectancy, lower fertility, later timing of births and higher levels of education are some of the most striking trends to occur during the demographic transition of economies. They are trends that account for major differences between developed and developing countries, and which may be seen as both causes and effects of changes in economic activity. As yet, there is no single theoretical model which is able to offer a unified explanation of these phenomena. In this paper we develop such a model, the hallmark of which is the joint, endogenous determination of longevity, child-bearing and human capital accumulation in dynamic general equilibrium.

Increases in life expectancy as an economy develops can be staggering, to say the least. Fogel (1994), for example, reports that, between 1850 and 1950, life expectancy at birth in the US rose by almost 75 per cent from 40 years of age to 68 years of age. By 1995, the figure had moved closer towards 100 per cent as average lifetime extended even further to 76 years of age (Statistical Abstract of the United States, 1995). Similar dramatic changes have taken place in other countries and are well-documented in numerous studies, including Easterlin (1996), Fogel (1997), Livi-Bacci (1997), the United Nations (1991) and the World Bank (1993). In addition, there is considerable evidence, surveyed and contributed to by Mirowsky and Ross (1998), to suggest that such changes are symptomatic of higher levels of education and human capital accumulation which not only make more resources available for spending on life-preserving activities (by raising standards of living and fostering economic growth), but which also encourage the adoption of healthy lifestyles on the part of individuals for various socio-economic reasons. Indeed, it is possible to argue that personal education improves personal health primarily because it improves personal effective agency: that is, education allows people to develop knowledge, skills and abilities that make them better equipped to create a way of living that is conducive to their welfare and that is not mediated by economic status.[1]

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[1] For example, the evidence presented by Mirowsky and Ross (1998) supports strongly the notions that better educated people are more able to coalesce health-producing behaviour into a coherent lifestyle, are more motivated to adopt such behaviour by a greater sense of control over the outcomes in their own lives, and are more likely to inspire the same type of behaviour in their children. As the same authors also point out, the

At the same time as lifetimes have lengthened, there have been noticeable shifts in fertility patterns which reveal not only a fall in birth rates but also a growing tendency for births to occur later, rather than sooner, in life. The historical decline in fertility is probably the most well-known stylised fact of demographic transition (see, e.g., Barro and Sala-i-Martin, 1995; Coale and Watkins, 1986; Dyson and Murphy, 1985; Wrigley, 1969; World Bank, 1984). That the timing of fertility decisions has altered as well is also corroborated by a large body of evidence, as surveyed by Hotz *et al*. (1997). Some straightforward calculations, for example, show that, during the past 30 years (both in the US and elsewhere), the probability of having a first child at age 20 has steadily fallen, while the probability of having a first child at age 35 has steadily risen. Like the decrease in mortality, both this intertemporal substitution of child-bearing and the general decline in fertility have often been studied in conjunction with observations about human capital accumulation. Of particular note has been the identification of a significant positive (negative) relationship between the timing of first births (number of births) and the length of time spent in education (see, e.g., Kravdal, 1990; Martinelle, 1990; Matthews *et al.*, 1982; Rindfuss *et al*., 1996).

At the theoretical level, there exists a growing class of dynamic general equilibrium models which attend separately to one or more, though not all, of the above observations. Barro and Becker (1989) and Becker and Barro (1988) present the seminal analysis of fertility choice and growth, treating both the timing of births and lifetimes as given, and abstracting from human capital accumulation.[2] Blackburn and Cipriani (1998) and Becker *et al.* (1990) extend this framework to allow for endogenous mortality and human capital accumulation, respectively. Galor and Stark (1993) introduce uncertain, but exogenous, lifetimes, along with human capital accumulation, while departing from endogenous fertility decisions. Ehrlich and Lui (1991) develop a similar model with fertility choice, but not the timing of fertility choice, re-included. And Iyigun (1996) considers the case of the joint determination of child-bearing, the timing of child-bearing and human capital accumulation, though still not the determination of lifetimes. Individually, these, and other,

direction of causation between better education and better health runs predominantly from the former to the latter, and not *vice versa*.

[2] See also Pavilos (1995) and Raut and Srinivasan (1994).

contributions are able to explain some, but not all, of the above evidence on demographic transition. The objective of the present paper is to develop a simple analytical model that is able to do so.[3]

We consider an overlapping generations economy in which the life expectancy of agents extends probabilistically to three periods. Agents are bearers of children, investors in education and producers and consumers of output. Child-bearing is costly in the sense that time must be spent on non-productive child-rearing activities, while educational investment is the means of accumulating human capital which raises the future productivity of labour. An exogenous increase in life expectancy (i.e., the probability of surviving for three periods) raises the opportunity costs of current work and reproduction by raising the future returns to human capital accumulation. Under such circumstances, agents devote more of their time to education and have fewer numbers of children when young, implying a higher growth rate of output and a lower growth rate of population. The innovation of our analysis is to endogenise life expectancy by allowing the probability of survival to depend on the level of development of the economy itself. That this may have important implications was evident to Hammermesh (1985) who also made the observation that individuals do, indeed, tend to extrapolate past improvements in longevity when determining their expected lifetimes (and, on average, tend to predict their horizons rather well).[4] Extending our model in this way not only allows us to account for all of the above facts about demographic transition, but also has the effect of creating multiple development regimes such that the limiting outcomes of the economy depend critically on initial conditions. As development now takes place, there is an increase in life expectancy, an increase in education, a decrease in fertility and a decrease in child-bearing early on in life. This process of transition is not smooth, however, there being a threshold level of capital, below which the economy is on a low development path and

[3] One aspect that we do not consider is the spacing of births. There is evidence to suggest that the interval between births is negatively related to a mother's initial stock of human capital (see., e.g., Nerlove and Razin, 1981) and that births tend to be clustered together rather than evenly spaced (see, e.g. Cigno and Ermisch, 1989). Incorporating these features would be an interesting extension to our model.

[4] The conclusion of this author was that "studies of life-cycle saving, investment in human capital and labour supply ignore changing life expectancy and its effect on subjective horizons and survival probabilities at the

above which the economy is on a high development path. Correspondingly, there is a low steady state equilibrium in which life expectancy is low, education is low, fertility is high and early child-bearing is high, and a high steady state equilibrium (or even an equilibrium with positive long-run growth) in which life expectancy is high, education is high, fertility is low and early child-bearing is low. These results complement those obtained in certain other models of fertility and growth (see, e.g., Becker *et al*., 1990; Galor and Weil, 1990; Iyigun, 1995; Nelson, 1956; Pavilos, 1995; Raut and Srinivasan, 1994), as well as those in the broader literature on poverty traps and threshold externalities. They are also related to the growing body of work on the development of economies over the very long-run and the transition from pre-industrial to post-industrial societies (see, e.g., Galor and Weil, 1998; Kremer, 1993; Jones, 1999; Tamura, 1996, 1999).[5]

The model is set out in Section 2. In Section 3 we solve for the optimal decisions of individuals. Section 4 contains our analysis of growth and demographic transition. Concluding remarks appear in Section 5.

2. The model

Time is discrete and indexed by $t = 0,\dots,\infty$. There is an endogenous population of reproductive agents belonging to overlapping generations with finite but uncertain lifetimes. Each agent matures safely through two periods of life and has a probability of surviving for a third period. After being raised by her parent in the first period of life, an agent becomes active as an investor in education, a bearer of children and a producer and consumer of output. Education is undertaken during the second period of life, while child-bearing, production and consumption take place during both the second and third periods. All agents have identical preferences and technologies, and are aware of

expense of realism with the possible price of incorrect behavioural implications". (Hammermesh, 1985, p.406.)

[5] In Tamura (1996), for instance, it is shown that, conditional on some growth occurring, it is possible to switch from a low development and high fertility regime to a high development and low fertility regime. The key ingredients of that analysis are a rising rate of return to human capital investment and a conditional externality effect in human capital investment such that only those agents who inherit a human capital stock greater than some threshold level are able to exploit the body of knowledge in society as a whole. What is not explained, however, is why this group of agents should inevitably choose growth to begin with (and why the other group of agents should inevitably not). The present paper may be seen as offering one possibility.

their life expectancies.

The expected lifetime utility of an agent of generation *t*, U^t, is given by

$$U^t = \log(c_{t+1}^t) + \gamma \log(n_{t+1}^t) + \pi_{t+1}^t \theta[\log(c_{t+2}^t) + \gamma \log(n_{t+2}^t) + \psi \log(s_{t+2}^t)], \quad \gamma, \psi > 0; \theta \in (0,1) \tag{1}$$

where c_{t+i}^t and n_{t+i}^t (i = 1,2) denote consumption and the number of children, respectively, s_{t+2}^t denotes leisure and π_{t+1}^t is the probability of surviving to the third period. As elsewhere, we simplify the analysis by making the following assumptions. First, parents are non-altruistic in the sense that they do not derive utility from the welfare of offspring, but merely from the production of offspring. Second, children are treated as consumption goods, yielding utility to their parents only during the period in which they are born. And third, both consumption and children are substituted intertemporally with unit elasticity. These assumptions make the analysis more tractable than it would otherwise be without causing much loss of generality.[6]

Each agent enters her second period of life with a given amount of human capital, part of which, $\underline{h}$, is derived from innate potential and part of which, h_{t+1}^{t-1}, is inherited from her parent. In general, of course, the concept of human capital need not be restricted to including just technical knowledge and skills, but may be broadened to encompass other personal attributes (health being the most notable in the present context) as well. Whatever the interpretation, an agent is understood to combine her initial human capital with labour, l_{t+1}^t, to produce output, y_{t+1}^t, according to

$$y_{t+1}^t = A(h_{t+1}^{t-1} + \underline{h})l_{t+1}^t, \quad A > 0 \tag{2}$$

The total time available to an agent is normalised to one and is exhausted on working, schooling and child-rearing. Following Becker *et al*. (1990) and others, we assume that it takes a fixed amount of time, $q \in (0,1)$, to raise each child. Time spent on education is therefore $1 - l_{t+1}^t - qn_{t+1}^t$. Given this, together with her human capital endowment, an agent produces her own human capital, h_{t+2}^t, in accordance with

[6] For example, our main results would be unchanged if we were to treat children as durable goods and allow parents to derive future utility from current offspring (by including a term such as $\pi_{t+1}^t \theta\delta \log(n_{t+1}^t)$ in (1), where $\delta < \gamma$ perhaps).

$$h_{t+2}^{t} = B(h_{t+1}^{t-1} + \underline{h})(1 - l_{t+1}^{t} - qn_{t+1}^{t}), \quad B \geq 1 \tag{3}$$

If an agent survives to the third period, then she no longer invests in education but allocates her time between working, child-rearing and leisure. The fraction of time available for working is $l_{t+2}^{t} = 1 - s_{t+2}^{t} - qn_{t+2}^{t}$ which the agent now combines with her new stock of human capital to produce y_{t+2}^{t} units of output in the same way as before:

$$y_{t+2}^{t} = Ah_{t+2}^{t}(1 - s_{t+2}^{t} - qn_{t+2}^{t}) \; . \tag{4}$$

The model is completed by specifying its most important feature which is the endogenous determination of the survival probability, π_{t+1}^{t}. It is this feature that accounts for our main results and which distinguishes our analysis from the existing literature. As mentioned earlier, it has been argued by many observers that changes in life expectancy owe much to changes in public awareness and personal lifestyles brought about unintentionally by changes in levels of education through which human capital is accumulated. This view of events invites a simple and tractable characterisation of π_{t+1}^{t} that will form the basis of most of our analysis. Of course, one may also think of π_{t+1}^{t} as being influenced by a number of other factors as well, both internal and external to an agent. These factors might range from private expenditures of income, time and effort (e.g., on medical treatment, hygiene and exercise), to government provided services and the quality of the environment (such as the extent of public health care, sanitation and pollution). In an Appendix we attend to such considerations explicitly, focusing, in particular, on the case in which π_{t+1}^{t} is determined initially by purposeful public policy (which is arguably the most relevant consideration, especially for developing countries). Compared to our main body of analysis, this extension of the model provides additional microfoundations but has no bearing on the marginal decisions of individuals (and leads ultimately to the same type of reduced form expression for π_{t+1}^{t}). For this reason, we find it convenient to focus on those decisions first and to postpone further discussion of π_{t+1}^{t} until later.

3. Individual fertility, education and production

An agent is faced with the problem of maximising (1) subject to (2), (3) and (4), together with $y_{t+i}^{t} = c_{t+i}^{t}$ (i = 1,2). Solving this problem for any given π_{t+1}^{t} yields the following optimal decision rules for labour supply and child demand:

$$l_{t+1}^{t} = \frac{1}{1+\gamma+\pi_{t+1}^{t}\theta}, \qquad l_{t+2}^{t} = \frac{1}{1+\gamma+\psi}, \tag{5}$$

$$n_{t+1}^{t} = \frac{\gamma}{q(1+\gamma+\pi_{t+1}^{t}\theta)}, \qquad n_{t+2}^{t} = \frac{\gamma}{q(1+\gamma+\psi)}. \tag{6}$$

The decision rule for education follows as

$$1 - l_{t+1}^{t} - qn_{t+1}^{t} = \frac{\pi_{t+1}^{t}\theta}{1+\gamma+\pi_{t+1}^{t}\theta}. \tag{7}$$

These decision rules depend on the parameters q (the cost of child-rearing), γ (the utility weight on offspring), ψ (the utility weight on leisure) and θ (the discount factor) in the ways that one would expect. Of greater interest is the dependence on π_{t+1}^{t}, the probability of life extension. An increase in this probability reduces both the supply of labour and demand for children in the second period of life, thereby increasing the amount of education in that period. Intuitively, a higher life expectancy raises the opportunity costs of current work and reproduction by raising the future return to human capital accumulation. As such, an agent allocates less of her current time to manufacturing and child-rearing, and more of this time to schooling. By virtue of the structure of the model, an agent’s choices in the third period are unaffected. This means that variations in life expectancy cause not only absolute changes within a period, but also relative changes between periods. In particular, an increase (decrease) in life expectancy implies a decrease (increase) in the demand for children early on in life relative to the demand for children later on in life. It is worth noting that this intertemporal shift in fertility patterns would survive in more general versions of the model, where third period choices are not necessarily fixed. In fact, the most natural generalisations that we can think of would make the effect more pronounced by admitting a direct substitution between child-bearing in one period and child-bearing in another.[7] Abstracting from this makes our subsequent analysis more straightforward without detracting from the basic result that changes in life expectancy cause changes in the intertemporal profile of fertility choice.

An increase in life expectancy reduces the total fertility of an agent, or family size, whether she survives to the third period or not. It would also reduce expected fertility (i.e., $n_{t+1}^t + \pi_{t+1}^t n_{t+2}^t$) if $\theta(1+\gamma+\psi) > (1+\gamma+\pi_{t+1}^t\theta)^2$. Since π_{t+1}^t is bounded from above (as well as below), this condition will always be satisfied if it is satisfied when π_{t+1}^t is at its maximum value. In turn, this will be more likely for higher values of ψ, the utility weight on leisure. A relatively strong preference for leisure during the later years of life is typically what one would presume.

Naturally, an increase in π_{t+1}^t has a positive effect on human capital accumulation by increasing the amount of time devoted to education. This is easily verified by combining (3) and (7) to obtain

$$h_{t+2}^t = \frac{B(h_{t+1}^{t-1} + \underline{h})\pi_{t+1}^t\theta}{1+\gamma+\pi_{t+1}^t\theta}. \tag{8}$$

This expression is the key to generating growth in the model and is the focus of the remainder of our analysis.

4. Demographics and development

The expression in (8) describes the equilibrium path of development of the economy. Changes in life expectancy cause changes in this path, affecting both the transitional dynamics and steady state of the economy. This result is notable as it stands, but it is all the more significant when one allows for the endogenous determination of life expectancy itself, as we do shortly.

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[7] For example, suppose that children were treated as durable goods and that parents derived end-of-life utility from the total number of offspring. In terms of (1), this would mean replacing the term $\gamma\log(n^t_{t+1})$ by a term such as $\gamma\log(n^t_{t+1} + n^t_{t+2})$. The demand for children in the third period would then be given by $(\gamma - qn^t_{t+1})/q(1 + \gamma + \psi)$, implying that an increase in π^t_{t+1}, which reduced n^t_{t+1}, would raise n^t_{t+2}. A similar result would be obtained if one assumed that the costs of raising children extended beyond one period. If parents devoted q' units of time in the third period to each of their children born in the second period, then the third period demand for children would now be $(\gamma - \gamma q' n^t_{t+1})/q(1 + \gamma + \psi)$.

If the probability of survival was constant, $\pi_{t+1}^{t} = \pi$ for all *t*, then there would be a unique equilibrium in which the economy would either converge to a steady state where long-run growth is zero, or develop perpetually along a constant positive growth path, depending on whether $B\pi\theta/(1+\gamma+\pi\theta) \in (0,1)$ or $B\pi\theta/(1+\gamma+\pi\theta) > 1$.[8] For given values of other parameters, an increase in π both raises the steady state equilibrium and makes the latter condition more likely to be satisfied. The reason, of course, is that a higher life expectancy motivates agents to invest more time in education and accumulate human capital. In doing so, agents have fewer children early on life and fewer children overall. Thus the model predicts that exogenous changes in life expectancy lead to changes in patterns of demography and development which accord well with the empirical evidence.

As indicated above, the implications of the model become more interesting still when we allow the probability of survival to be endogenous. This is the main innovation of our analysis and gives rise to the possibility of multiple development regimes. To repeat the point made initially, one of the most striking aspects of demographic transition is the rise in life expectancy: the more that an economy develops, the more that agents expect to live longer. An immediate and plausible way of capturing this would be to assume that each agent's probability of survival is an increasing (though bounded) function of the stock of human capital that she inherits from her parent - that is, $\pi_{t+1}^{t} = \pi(h_{t+1}^{t-1})$, where $\pi'(\cdot) > 0$, $\pi(0) = \underline{\pi}$ and $\lim._{h\to\infty} \pi(\cdot) = \overline{\pi} \leq 1$. This would be consistent with some of the ideas expressed earlier and with the findings of numerous empirical studies which testify to strong positive correlations between parental education and various measures of life expectancy, such as infant mortality and morbidity, adult frailty and infirmity, and the postponement of fatality risks to later stages of life (see, e.g., Becker 1998; Bishai 1996; Leigh 1998; Mirowsky and Ross 1998; Redman *et al*. 1992; Sandiford *et al*. 1995). Other supporting evidence can be found in more specific investigations, such as those of Cooksey *et al*. (1996), Flay *et al*. (1994), Greenlund *et al*. (1996) and Kandel and Wu (1995) who observe that better educated

[8] The steady state level of human capital in the absence of long-run growth is $B\pi\theta\underline{h}/(1+\gamma+(1-B)\pi\theta)$.

parents tend to have children who are less likely to take up smoking, less likely to become overweight and less likely to be sexually promiscuous.[9] An alternative approach would be to model life expectancy on the basis of microfoundations that focus on purposeful and costly actions undertaken by individuals and society. As mentioned previously, we do this in an Appendix with a formal illustration of how π_{t+1}^{t} might depend initially on factors that lie within the realm of public policy. Compared with the foregoing treatment, the upshot of that analysis is the same type of reduced form relationship between π_{t+1}^{t} and h_{t+1}^{t-1}, as summarised by the function $\pi(\cdot)$.

Given the above, expression (8) is now understood to define a transition function, $G(\cdot)$, such that

$$h_{t+2}^{t} = G(h_{t+1}^{t-1}) = \frac{B(h_{t+1}^{t-1} + \underline{h})\pi(h_{t+1}^{t-1})\theta}{1+\gamma+\pi(h_{t+1}^{t-1})\theta}, \tag{9}$$

where $G'(\cdot) > 0$ and $G''(\cdot) \gtreqless 0$. A steady state equilibrium is a fixed point of this mapping, $h^{*} = G(h^{*})$, and is stable (unstable) if $\lim._{h\to h^{*}-} G(\cdot) > (<)\, h^{*}$ and $\lim._{h\to h^{*}+} G(\cdot) < (>)\, h^{*}$. A steady state that is stable entails zero long-run growth, while a steady state that is unstable admits the possibility of positive long-run growth.

The existence of multiple development regimes means that the limiting outcomes of the economy are non-ergodic but depend crucially on initial conditions. The clearest illustration of this is provided by the case in which $\pi(\cdot)$ takes on the form of a simple step function,

$$\pi(h_{t+1}^{t-1}) = \begin{cases} \overline{\pi} & if \quad h_{t+1}^{t-1} \geq h_c \\ \underline{\pi} & if \quad h_{t+1}^{t-1} < h_c \end{cases}, \tag{10}$$

for some critical level of human capital, $h_c > 0$. The transition function may then be written as

$$G(h_{t+1}^{t-1}) = \begin{cases} \overline{g}(h_{t+1}^{t-1}) = \dfrac{B(h_{t+1}^{t-1} + \underline{h})\overline{\pi}\theta}{1+\gamma+\overline{\pi}\theta} & if \quad h_{t+1}^{t-1} \geq h_c \\ \underline{g}(h_{t+1}^{t-1}) = \dfrac{B(h_{t+1}^{t-1} + \underline{h})\underline{\pi}\theta}{1+\gamma+\underline{\pi}\theta} & if \quad h_{t+1}^{t-1} < h_c \end{cases}, \tag{11}$$

[9] By adopting a broad concept of human capital, one may also think in terms of the well-established positive association between parental health (and perhaps family background, in general) and child health. That initial

where $\overline{g}(0) > \underline{g}(0)$ and $\overline{g}'(\cdot) > \underline{g}'(\cdot)$. Figure 1 depicts the possible outcomes, where we assume that $\underline{g}'(\cdot) \in (0,1)$. An economy with an initial human capital stock below h_c converges to a low steady state, h_L^*, while an economy with an initial human capital stock above h_c either converges to a high steady state, h_H^*, or grows perpetually at a constant rate depending on whether $\overline{g}'(\cdot) \in (0,1)$ or $\overline{g}'(\cdot) > 1$. The initial stock of human capital now determines the initial probability of survival and, with it, the initial allocation of resources between education, working and child-rearing. Given these outcomes, which subsequently remain unchanged, the economy develops along one of two paths, either to the left or to the right of h_c. To the left of h_c, the economy is on a low development path, where life expectancy is low, education is low, fertility is high and early child-bearing is high. To the right of h_c, the economy is on a high development path, where life expectancy is high, education is high, fertility is low and early child-bearing is low. Thus, depending on initial conditions, the economy is predicted either to stagnate or to prosper, and to display the type of demographic behaviour in each case that one would typically expect.

The same results can be obtained if we allow $\pi(\cdot)$ to take on a more general form that is smooth rather than discontinuous. Although the analysis is slightly less straightforward, the model yields a more complete picture of events by permitting both the probability of survival and the allocation of resources to vary during the process of transition. Naturally, the transition function, $G(\cdot)$, is also continuous in this case and the properties of this function are studied in an Appendix. Given the restrictions on $\pi(\cdot)$, these properties are indeed such as to be capable of generating the same type of multiplicity of regimes as above. We show this in Figure 1, where our portrayal of $G(\cdot)$ is deliberately stylised for illustrative purposes.[10] As the economy now develops, there is an increase in life expectancy, an increase in educational investment, a decrease in family size and a decrease in child-bearing early on in life (all of which, of course, are the stylised facts of demographic

conditions and circumstances are important for future well-being is an established fact as well. For recent discussions of these, and related, issues, see Mirowsky and Ross (1998) and Smith (1999).

transition). But at what level of development the economy will end up depends critically on what level of development it starts off at: as above, poverty or prosperity to begin with implies poverty or prosperity in the future.

Of course, the prospective fortunes of an economy may change with changes in circumstances, whether by accident or design. Thus, for a given threshold level of development, h_c, exogenous shifts in the stock of human capital (or the initial stock of human capital) may cause a switch in development regime by pushing the economy either above or below that threshold. Likewise, for a given stock of human capital, changes in the values of structural parameters (e.g., shifts in human capital and health technologies) can produce a similar turn of events by altering the transition function, $G(\cdot)$, and the threshold, itself. In both cases it is clear that a switch in regime is more likely to occur the closer is an economy to h_c to begin with. This suggests that, should circumstances change for the better, it is those countries at the upper end of the distribution below h_c that stand a greater chance of launching onto the high development path, while those at the lower end of the distribution are liable to be left behind. In addition, should different countries face different parameter configurations to begin with, then one would observe cross-country differences in transition functions that may hold little prospects for cross-country convergence. That is, there would be a distribution of development paths and a corresponding distribution of long-run outcomes. Whichever way one looks at it, the divisions between poor countries and rich countries are unlikely to vanish quickly or easily, if at all, in our model.

Our analytical results are confirmed by numerical simulations of a calibrated version of the model under alternative specifications of $\pi(\cdot)$. One simple, but flexible, specification that we use for illustration is

$$\pi(h_{t+1}^{t-1}) = \frac{\underline{\pi} + \overline{\pi}\Phi(h_{t+1}^{t-1})^{\phi}}{1+\Phi(h_{t+1}^{t-1})^{\phi}}, \quad \Phi, \phi > 0 \tag{12}$$

[10] Clearly, the precise shape and position of this function (and therefore both the critical and steady state levels of capital) will depend on the particular form of $\pi(\cdot)$, together with the particular values of the parameters in the model.

which satisfies the restrictions $\pi'(\cdot) > 0$, $\pi(0) = \underline{\pi}$ and $\lim_{h\to\infty} \pi(\cdot) = \overline{\pi}$, as well as displaying the property $\pi''(\cdot) \gtreqless 0$ for $h_{t+1}^{t-1} \lesseqgtr [(\phi - 1)/\Phi(\phi + 1)]^{1/\phi}$.[11] We focus on the case in which the high development regime is characterised by transitional dynamics towards a balanced, endogenous growth path. Treating each period as 25 years, our baseline parameter values are A = 1.00, B = 9.00, $\underline{h}$ = 0.10, q = 0.70, γ = 2.00, ψ = 2.50, θ = 0.62, $\underline{\pi}$ = 0.30, $\overline{\pi}$ = 0.95, ϕ = 4.00 and Φ = 0.01. Along the balanced growth path, these values imply an annual discount factor of 0.98, an annual population growth rate of 1%, an annual per capita income growth rate of 1.6%, an average allocation of adult time to working of 25% and a life expectancy of 74 years. The low steady state equilibrium occurs at h_L^* = 0.11, where life expectancy is 57 years, while the threshold point occurs at h_c = 5.35, where life expectancy is 65 years. Given these outcomes, an economy that is close to h_L^* would require an extremely large (50-fold) increase in its stock of human capital to jump just beyond h_c into the high development regime, implying a non-trivial (8 year) leap in the life expectancy of its citizens.

Among various parameter changes that one might consider, the most interesting in the present context are changes in the parameters of the human capital production and mortality functions. In Table 1 we summarise the results of our numerical experiments with variations in the shift factors B and $\underline{\pi}$. An increase in the value of either of these parameters has the effect of pulling up the transition function, $G(\cdot)$, such that h_L^* is raised while h_c is lowered. The effect on life expectancy in the low steady state equilibrium is negligible when B is increased but positive when $\underline{\pi}$ is increased. Conversely, the effect on per capita income growth along the balanced growth path is

[11] Thus, if $\phi \in (0,1)$, then $\pi(\cdot)$ is strictly concave for all h, while if Φ = 0, then $\pi(\cdot) = \underline{\pi}$ for all h. More generally, the parameters ϕ and Φ jointly determine both the turning point in $\pi'(\cdot)$ and the speed at which $\pi(\cdot)$ traverses the interval $(\underline{\pi}, \overline{\pi})$. For a given value of ϕ (Φ), an increase (decrease) in Φ (ϕ) reduces the turning point, while for a given value of such a point, an increase (decrease) in Φ (ϕ) raises the speed of transition (the limiting case of which is when $\pi(\cdot)$ changes value from $\underline{\pi}$ to $\overline{\pi}$ instantaneously, which corresponds to the case of a step function). An example of an alternative specification with broadly the same implications is the logistic function $\pi(h) = \{\underline{\pi}[\exp(\Delta\delta) + 1] + \overline{\pi}[\exp(\Delta h) - 1]\}/[\exp(\Delta\delta) + \exp(\Delta h)]$, for which the turning point and speed of transition are determined independently by the parameters δ and Δ, respectively. We have also confirmed our results in other simulations for the case in which $\pi(\cdot)$ is derived according to the type of framework presented in Appendix B.

positive when B is increased but zero when $\underline{\pi}$ is increased. For example, if B = 12.00 (15.00), an economy in the high development regime would converge to an equilibrium with a balanced growth rate of 2.75% (3.67%), while if $\underline{\pi}$ = 0.40 (0.50), an economy in the low development regime would converge to an equilibrium with a life expectancy of 60 years (63 years). For B > 15.90, or $\underline{\pi}$ > 0.56, the transition function lies everywhere above the 45^0 line and the multiplicity of development regimes vanishes. Extending this analysis, one may also study how similar differences in circumstances might translate into differences in populations of poor and rich countries. To take the simplest illustration, consider an initial state of affairs in which the world stock of human capital is uniformly distributed over a continuum of economies, a unit of mass of which is located within the interval $(0, h_c)$ at our benchmark parameter configuration. As one increases the value of either B or $\underline{\pi}$, this mass decreases as the interval shrinks. Thus, using the same examples as above, if B = 12.00 (15.00), or $\underline{\pi}$ = 0.40 (0.50), there would be 26% (50%), or 10% (27%), fewer countries assigned to the low development regime.

While the model is deliberately stylised for the usual reasons, the foregoing results are both instructive and revealing. On the one hand, they indicate how small shifts in technology or life expectancy can tip the balance in favour of prosperity (or industrial revolution, perhaps) for countries just below the threshold level. On the other hand, they demonstrate how larger shifts in such factors (in particular, increases in $\underline{\pi} \leq 0.56$) can cause countries at the lower end of the distribution to undergo demographic transition (extensions of life expectancy) without experiencing economic development (changes in their destiny away from h_L^*). This last feature is particularly noteworthy, given the observed behaviour of actual economies. In Easterlin (1996), for example, it is estimated that life expectancy at birth in the relatively more developed (less developed) regions increased from around 66 (41) years to 74 (61) years between 1950-55 and 1985-90, and is set to reach the order of 79 (71) years by 2020-25. The generally-accepted interpretation of these trends is summarised succinctly by the same author: "The much more rapid spread of the mortality revolution *vis-à-vis* modern economic growth chiefly reflects the fact that the institutional, physical capital and educational requirements for the technology of disease control are considerably less than those for the modern technology of economic production. As a result, the mortality revolution has occurred in countries with low, and even stagnating or declining, real per capita income, and life expectancy differentials throughout the world, unlike those in per capita income, are converging rapidly." (Easterlin, 1996, p.81.) According to our model, increases in $\underline{\pi}$ (representing widespread advances in health technology) can, indeed, lead to improvements in life expectancy without eliminating poverty traps. Only when $\underline{\pi}$ exceeds a certain a value does the multiplicity of development regimes vanish. From a more optimistic perspective, this might be taken to suggest that as advances in health technology continue (coupled, perhaps, with the slower diffusion of production technologies), it is only a matter of time before the less developed economies make the final transition to economic development.

5. Conclusions

It is natural to presume that life expectancy is an important factor in determining the life-cycle behaviour of individuals. At the same time, it is unnatural to presume that life expectancy is wholly exogenous and independent of economic conditions. Until now, models of fertility choice and growth have been based on both presumptions, meaning that they have given part, but not the whole, of the picture. The model developed in this paper is a first attempt at filling in some of the gaps by allowing for endogenous lifetimes and two-way causality in the relationship between longevity and economic activity. Incorporating these features, together with the other notable aspect of intertemporal fertility substitution, not only brings the theory closer to reality but also yields additional insights into the process of demographic transition. As well as being able to explain a number of empirical observations, our analysis has important implications for the long-term development of an economy and the extent to which initial inequalities between poor and rich countries are likely to persist. These implications are complementary to those found in the existing literature on poverty traps, growth miracles and threshold externalities, but are derived from a different perspective that sheds new light on the issue of why some countries may permanently lag behind others. On the basis of our results, we view our analysis as a promising first step in untangling the fertility-longevity-development nexus.

Appendix A. Generalised $\pi(\cdot)$

The transition function, $G(\cdot)$, is defined by (9), from which we obtain

$$G'(\cdot) = \frac{B\theta[(1+\gamma+\pi(\cdot)\theta)\pi(\cdot) + (1+\gamma)(h+\underline{h})\pi'(\cdot)]}{(1+\gamma+\pi(\cdot)\theta)^2} > 0\,, \tag{A.1}$$

$$G''(\cdot) = \frac{B\theta(1+\gamma)\{2(1+\gamma+\pi(\cdot)\theta)\pi'(\cdot) + (h+\underline{h})[(1+\gamma+\pi(\cdot)\theta)\pi''(\cdot) - 2\theta(\pi'(\cdot))^2]\}}{(1+\gamma+\pi(\cdot)\theta)^3} \gtreqless 0\,, \tag{A.2}$$

where $\pi(\cdot)$ satisfies $\pi'(\cdot) > 0$, $\pi(0) = \underline{\pi}$ and $\lim._{h\to\infty} \pi(\cdot) = \overline{\pi} \le 1$. Since there must exist an $\tilde{h}$ for which $\pi''(\cdot) < 0$ for all $h > \tilde{h}$, then $\lim._{h\to\infty} h\pi'(\cdot) = 0$ so that $\lim._{h\to\infty} G'(\cdot) = B\overline{\pi}\theta/(1+\gamma+\overline{\pi}\theta)$ and $\lim._{h\to\infty} G''(\cdot) = 0$. As in the case of a step function, therefore, long-run growth is either zero or positive according to whether $B\overline{\pi}\theta/(1+\gamma+\overline{\pi}\theta) \in (0,1)$ or $B\overline{\pi}\theta/(1+\gamma+\overline{\pi}\theta) > 1$.

A fixed point of the transition mapping satisfies $h^* = G(h^*)$, or

$$J(h^*) = K(h^*)\,, \tag{A.3}$$

where $J(h^*) = (1+\gamma)h^*$ and $K(h^*) = \theta[(B-1)h^* + B\underline{h}]\pi(h^*)$. Evidently, $J'(\cdot) > 0$, $J''(\cdot) = 0$, $K'(\cdot) > 0$ and $K''(\cdot) \gtreqless 0$. In addition, $J(0) = 0$ while $K(0) > 0$. A fixed point is locally stable if $G'(h^*) \in (0,1)$ which is equivalent to requiring

$$J'(h^*) > K'(h^*)\,. \tag{A.4}$$

Sufficient conditions for a unique, stable equilibrium are that $B\overline{\pi}\theta/(1+\gamma+\overline{\pi}\theta) \in (0,1)$ and $\pi(\cdot)$ is strictly concave. Under such circumstances, $K''(\cdot) < 0$ so that $K(\cdot)$ crosses $J(\cdot)$ only once and does so from above. If either or both of these conditions are not satisfied, however, then there may be more than one equilibrium which alternate between stability and instability. For example, if $B\overline{\pi}\theta/(1+\gamma+\overline{\pi}\theta) \in (0,1)$ but $\pi''(\cdot) > 0$ for all $h < \tilde{h}$, then $K''(\cdot) > 0$ for all $h < \tilde{h}$ as well, implying the possibility of an equilibrium triple $\{h_L^*, h_c, h_H^*\}$ such that $J'(h_L^*) > K'(h_L^*)$, $J'(h_c) < K'(h_c)$ and $J'(h_H^*) > K'(h_H^*)$. Additionally, if $B\overline{\pi}\theta/(1+\gamma+\overline{\pi}\theta) > 1$, then there is also the possibility of just the equilibrium pair $\{h_L^*, h_c\}$ which implies positive long-run growth for an economy that starts off with $h > h_c$.

Appendix B. $\pi(\cdot)$ from microfoundations

Suppose that society operates a fully-funded, balanced budget welfare programme, whereby each generation pays for its own chance of living beyond two periods by incurring flat-rate taxes on its income during the second period of its life in return for the provision of various services (medical care, sanitation, environmental improvement and the like) that contribute to its life expectancy. Let τ denote the tax rate (assumed, but not required, to be constant), X_{t+1}^t denote the total amount of life-preserving services supplied to agents of generation t and N^t denote the total population of such agents. The second period budget constraint for an agent is $c_{t+1}^t = (1-\tau)y_{t+1}^t$, while the budget constraint for the government is $X_{t+1}^t = \tau N^t y_{t+1}^t$. Assume that access to public services is subject to a congestion cost such that the actual amount of services available to each agent is $x_{t+1}^t = X_{t+1}^t \big/ N^t$. The probability of survival is specified initially as

$$\pi_{t+1}^t = \pi_0(x_{t+1}^t), \tag{B.1}$$

where $\pi_0'(\cdot) > 0$, $\pi_0(0) > 0$ and $\lim._{x\to\infty} \pi_0(\cdot) = \overline{\pi} \leq 1$. Since x_{t+1}^t is taken as given by each agent, and since τ has no effect on marginal decisions (due to logarithmic utility), the results in (5) - (8) remain unchanged. The government's budget constraint may therefore be written as

$$x_{t+1}^t = \frac{\tau A(h_{t+1}^{t-1} + \underline{h})}{1 + \gamma + \theta\pi_0(x_{t+1}^t)}, \tag{B.2}$$

which implies $x_{t+1}^t = x(h_{t+1}^{t-1})$, where $x'(\cdot) > 0$. Substitution into (B.1) delivers $\pi_{t+1}^t = \pi_0(x(h_{t+1}^{t-1})) = \pi(h_{t+1}^{t-1})$, where $\pi'(\cdot) = \pi_0' x' > 0$. If $\pi_0(\cdot)$ takes the form of a simple step function ($\pi_0(\cdot) = \underline{\pi}\,(\overline{\pi})$ for $x < (\geq)\, x_c$), then so too does $\pi(\cdot)$ ($\pi(\cdot) = \underline{\pi}\,(\overline{\pi})$ for $h < (\geq)\, h_c$), as in (10) of the main text. For the more general case, it is known that there must exist an $\tilde{x}$ such that

$\pi_0''(\cdot) < 0$ for all $x > \tilde{x}$. Since $\pi''(\cdot) = \pi_0'' x'^2 + \pi_0' x''$, it can then be verified that there must also exist an $\tilde{h}$ such that $\pi''(\cdot) < 0$ for all $h > \tilde{h}$, as in Appendix A.

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Figure 1. Multiple development regimes.

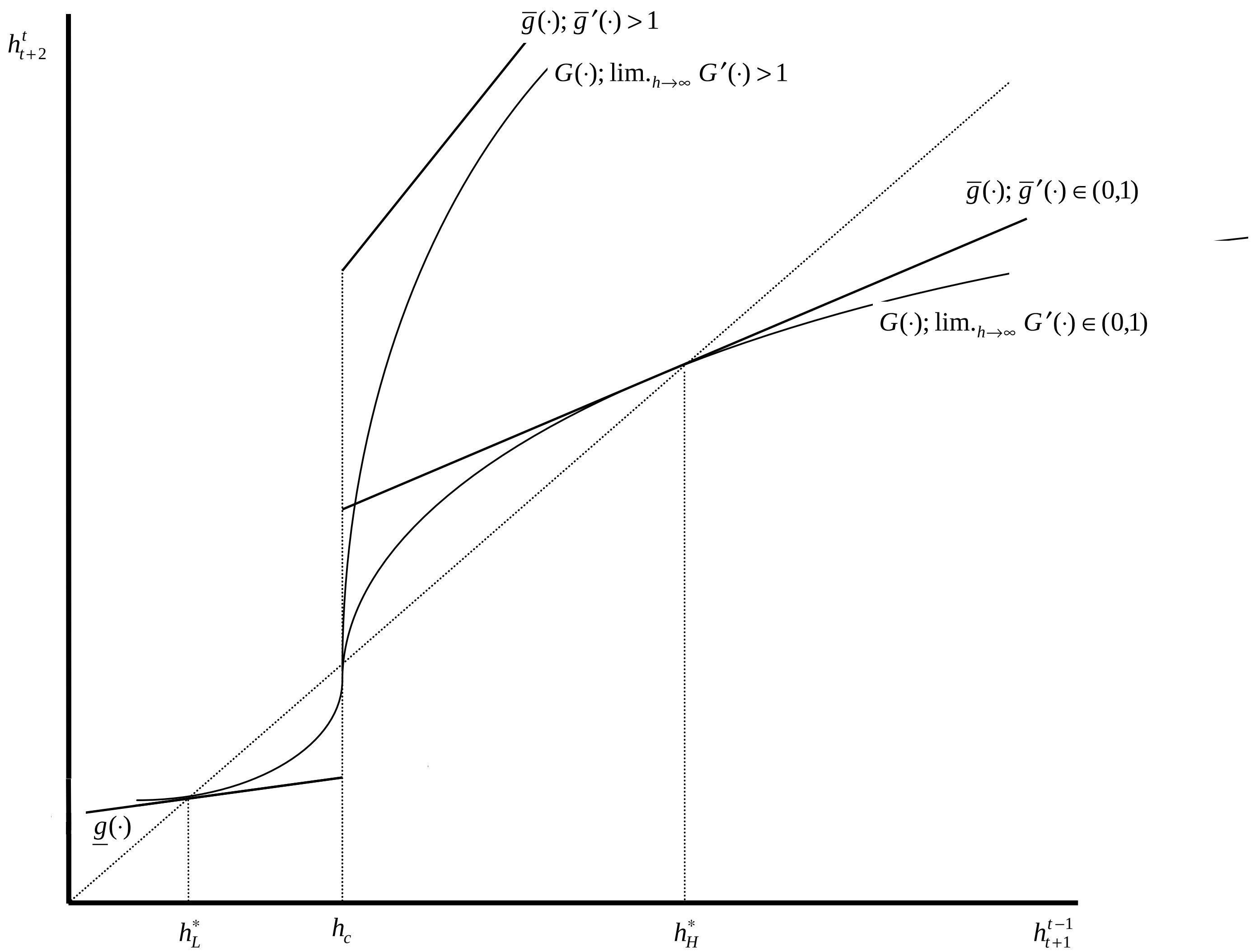

Table 1. Simulation results under alternative parameter values

B	$(\underline{\pi})$	h_L^*		h_c		Life expectancy at h_L^*		Balanced annual growth rate	
9.00	(0.30)	0.11		5.35		57		1.60	
10.50	(0.35)	0.16	(0.15)	4.57	(5.10)	57	(59)	2.21	(1.60)
12.00	(0.40)	0.23	(0.22)	3.96	(4.80)	57	(60)	2.75	(1.60)
13.50	(0.45)	0.37	(0.33)	3.38	(4.44)	57	(61)	3.24	(1.60)
15.00	(0.50)	0.71	(0.54)	2.69	(3.95)	57	(63)	3.67	(1.60)
15.90	(0.56)	1.52	(1.47)	1.76	(2.75)	58	(64)	3.92	(1.60)