Testing Inverse Projection, Differentiated Inverse Projection and Stochastic Inverse Projection: A Reconstruction of the Population of Sardinia between 1861 and 1921 Using Three Different Techniques

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1 Overall Objective of the Project, Data and Procedures Presentation

Twenty five years after the presentation of the first version of Inverse Projection (IP), the technique, conceived by Ronald Lee, has been utilitised as a powerful and reliable means of studying populations over time when only vital series are available. During this period, other authors have suggested some modifications to the original procedure without changing the basic logic. They have also tried to create more and more precise estimates by overcoming some conceptual and operative limitations found in Inverse Projection.

This short work is based on the use of three techniques belonging to the inverse projection class: the classic version by R. Lee [7, 8] and two of the most recent Italian solutions, namely the Differentiated Inverse Projection (IPD) by A. Rosina[1] and the Stochastic Forward Inverse Projection (SIP) by S. Bertino and E. Sonnino [1, 2][2]. The overall objective of this project is to compare the estimates produced by the three procedures using the same fundamental data (time series of births and deaths, initial age structure) and, whenever possible, the same mortality, fertility and migratory models.

[1] The first version of the technique conceived by Rosina (working with quinquennial data) is explained in Rosina [9, 10], Rosina and Rossi [13, 14]. The more recent version, which works with annual data, and is used in this work, is explained in Rosina [12]. The programme (called IPD3) created by Alessandro Rosina, which enables the procedure's execution, has been employed by us in the standard way (IP) as well as in the differentiated way (IPD). The latter enables the exploitation of the death distribution by age. In our work we have separated deaths at age 0 from deaths at age $1-\omega$.

[2] We thank Elisabetta Barbi for very helpful suggestions regarding the execution programme.

The suggested applications are concerned with Sardinia (a region in Italy) in two different periods: from 1862 to 1901 and from 1901 to 1921.

We now give some justification for the choices of the time intervals and of the population being considered (the Sardinia region reconstruction). When it comes to a reconstruction within the inverse projection logic, the possible distortions of the produced estimates are mainly due to three factors:

1) first of all, to the bad quality of the available data (i.e. underestimation or overvaluation problems concerning the basic data, initial age structure distortions, etc.);
2) secondly, to the non correspondence of mortality, fertility and migratory models within the situation under analysis;
3) lastly, to the fact that the demographic system under analysis can be particularly disturbed and that the available data do not reveal these disturbances. Accordingly, particular problems can take place in the presence of irregular and massive migration flows.

In order to clarify the comparison among the various procedures, we have tried to reduce the negative effect of the aforementioned factors to their lowest level. With reference to Italy, the period following its political union is quite rich in complete and reliable statistics.

In fact, at the regional level, vital series have been published in Italy since 1862, and census data (and therefore the age structure of the population) are available for the years 1861, 1871, 1881, 1901, 1911 and 1921. Abridged or complete life tables have been calculated (for males and females) at the regional level for 1871-72, 1881-82, 1900-01, 1910-12 and 1921-22[3]. Deaths by age are also available[4], for almost every year during the period 1863-1921.

Sardinia, (with a population of 588,000 inhabitants in 1861, increasing to 860,000 in 1921) represents a special case among Italian regions. Because it is an island, it has experienced no boundary changes (an issue which would cause data homogeneity problems) and it has also experienced only greatly reduced migratory flows, as shown in Table 1.

Migrations are almost negligible until 1900 and, due to this reason, we have decided to carry out the first reconstruction (by using the three procedures) on this population, considering both sexes combined, for the period

[3] The complete regional life tables (separate for males and females) for the period 1921-22 have been created by Gini and Galvani [6]. The other abridged life tables (also separated by sex), edited by L. Del Panta, are still unpublished (but they can be requested from the author). For those life tables covering the 1881-82 period (the adopted criteria are the same) see Del Panta [5].

[4] We ignore them as far as Inverse Projection and Stochastic Inverse Projection are concerned. However, we account for (for every year in the period 1862-1921) the number of deaths aged 0 and $1-\omega$, when using Differentiated Inverse Projection.

Table 1. Migration rates ‰ toward foreign countries from Italian regions, 1876-1913

Regions	1876-1880	1881-1890	1891-1900	1901-1910	1911-1913
Piedmont	9.1	9.9	8.0	16.5	19.0
Lombardy	5.0	5.8	5.0	11.3	15.7
Venetia	12.0	20.3	33.7	29.5	31.4
Liguria	5.0	6.0	3.8	6.1	7.0
Emilia-R.	1.9	3.0	5.6	12.9	13.2
Tuscany	3.3	4.8	5.8	11.9	15.3
Umbria	0.0	0.1	1.2	15.0	21.3
Marche	0.3	2.0	4.8	20.6	24.8
Latium	0.0	0.0	1.4	9.8	13.4
Abruzzi-M.	1.0	6.5	10.7	33.7	32.8
Campania	2.1	5.5	10.6	21.6	20.0
Apulia	0.3	0.8	1.8	10.7	14.2
Basilicata	6.0	16.5	18.2	29.8	29.2
Calabria	1.8	7.9	12.1	31.6	31.7
Sicily	0.3	1.7	5.0	21.5	26.2
Sardinia	0.0	0.2	0.9	6.9	10.4
ITALY	3.9	6.4	9.0	17.3	20.2

Source: Del Panta [4]

covering the two censuses from 1861 to 1901. We have then carried out a second reconstruction for the following twenty years (1901-1921), a period which included a severe crisis of mortality caused by the Spanish influenza and coinciding with the end of the First World War. One of the objectives of our paper is to compare the behaviour of the three techniques on the occurrence of a severe mortality crisis. In the presence of significant migratory movements we have decided to carry out this second reconstruction considering only the female sex, because, as seen from Table 2, this one is generally less involved in migratory flows.

Table 2. Male and female migration from Sardinia, 1901-1921

Periods	Males (1)	Females (2)	Total (3)	Sex Ratio (1) / (3)
1901-1910	494583	91224	585807	0.844
1911-1921	178717	76347	255064	0.700
1901-1921	673300	167571	840871	0.800

Source: Commissariato Generale dell'Emigrazione [3]

As far as the available data are concerned, note that the first three censuses (1861, 1871 and 1881) are affected by serious distortions in distribution per age because people were only asked their actual age and not their birth date. Therefore, there are strong concentrations in ages ending in zero, and they are more evident among adult and old people. In particular, the census from 31st December 1861, the first one after Italy's political union, appears to be particularly rich in error and distortions. For this specific reason, to create the first model structure (for the 1862-1901 period), we thought it was appropriate to use the Sardinia age structure obtained in the 1871 census because it was probably less affected by mistakes. We assumed that the situation mentioned above was not much different from the one which occurred ten years beforehand[5]. As far as the mortality models employed are concerned, they have been extracted (in the same way for the three procedures[6]) moving from the abridged tables 1871-72 and 1900-01 (for the projection regarding the 1862-1901 period), and from the tables 1900-01 and 1921-22 (for the projection concerning the 1901-1921 period)[7]. Finally, the same standard fertility and migration patterns were adopted for every reconstruction.

All things considered, by applying the three different procedures (IP, IPD, SIP) to the same data set, by using the same mortality, fertility and migratory models and the same initial age structure[8], we have created the optimal conditions to evaluate the "matching" of the different obtained reconstructions to real data. On the other hand, the structure by age obtained with these procedures can be compared to the one obtained by means of censuses. Similarly, during the years for which life tables have been built, it is possible to compare the estimated values of life expectancy at birth to those inferred from the tables based on the effective knowledge of distributions by age of deaths and population. On the contrary, it is not possible to make a similar comparison as far as the total fertility rate and gross reproduction rate are concerned, because there is no data available, nor is it at the national level, for births classified by women's age distribution, for the period under analysis.

[5] Also in this case, however, the previously mentioned disadvantage appears, so there has been made a smoothing in the structure by age by means of a series of interpolations.

[6] It is here necessary to remember that for IP and IPD it is necessary to provide L_x (number of years lived) series while for SIP it is important to provide q_x (probability of dying) series. SIP in fact, deals with classic probabilities of dying between exact durations, while IP and IPD use the probability of dying between completed durations. This difference, as we will see further on, brings some disparities in the results of the different procedures, mainly when there are strong fluctuations in yearly deaths.

[7] The passage from the abridged to the complete life tables (which is necessary for the "year period" projections) has been done by means of the Brass logit system.

[8] We remind the reader that SIP requires the initial population age structure divided by sex, also when only a reconstruction by combined sexes is required, as it happened for the 1862-1901 period.

For brevity's sake, we have confined the principal results to Appendix A, that is to say, the annual values of the two indicators which better characterise the demographic system: the life expectancy at birth (e_0) and the Gross Reproduction Rate (GRR). In the paper we present some very concise tables which contain the average decennial values of the indicators themselves.

Before commenting on the results, we want to point out that there was a problem regarding the projection made by using SIP for the second period, including the year 1918, which saw a considerable increase in mortality. In cases like that, the programme enables the insertion of a life table which is appositely made in order to be more similar to the structure by age of the mortality caused by the epidemic or whatever factor increased deaths. By knowing the distribution by age of deaths both in 1918 and in the other years of the projection (see data in Appendix B and Figure 5), we could state that the flu epidemic affected people in different ways according to age, partially excluding babies and old people, but being stronger with youngsters and children from two years old onwards. From these reflections we have tried to estimate a survival function despite the lack of data regarding the structure by age of the population in that year[9].

2 Results Regarding the 1862-1901 Period

In this first period, for people from Sardinia, the mortality decline phase takes place while at the same time, fertility increases until the first half of the 1880s, after which time it decreases until the beginning of the new century (see results for the following period). The projections have been made, as previously stated, by using the same models of mortality, fertility and migration. In the Differentiated Inverse Projection (IPD) deaths have been divided into classes like 0 and $1 - \omega$, while the reconstruction by means of the Stochastic Inverse Projection has been carried out with five simulations, a reduction coefficient equal to two, and by choosing a random distribution of births. Again, this is a study which takes both sexes into account and, considering the final results, here we start analysing those results concerning the expectation of life at birth.

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[9] We have adopted a very simple procedure (and also an arbitrary one), moving from the calculus, per each age, of the ratio between deaths which occurred in 1918 and the average number (of deaths) per year over a certain number of adjoining years. After obtaining directly, on the basis of real data (births and deaths 0-1), the q_0 value, as far as the following ages are concerned, the death probability values have been obtained by moving from those in the 1921-22 life table, accepting the hypothesis according to which deaths percentage variations are equivalent to q_x percentage variations. The level of life expectancy at birth obtained by means of this procedure is not so different from that estimated for the year 1918 from IP and IPD.

As already mentioned, the analytical results are present in Appendix A (see Figure 1 in particular), while Table 3 synthesises comparisons over ten-year periods. At that time, Sardinia was still concerned with the endemic presence of malaria, and this is the main reason for the low survival level which appears in the first ten years after the political union. As seen in the chart, in Sardinia for the period taken into consideration, there have not been exceptional mortality crises. With the trend of an increasing survival level, the high increases in life expectancy at birth in the periods 1866-67 and 1882-83 are followed by strong decreases as well. In any case, both Figure 1 and Table 3 show a high level of consistency between the results of the different procedures. When there are no heavy mortality crises which selectively affect the various age classes[10], IP and IPD provide similar e_0 estimates. From the penultimate column in Table 3, the difference among results appears to be fixed under 1 percent. Also the SIP estimates are not much different from the IP ones. The most striking differences belonging to the first decade (1862-70) are mainly due to a slightly irregular value estimated by IP for the first year of projection (see annual data in Appendix A) but also from the fact that, in the strong e_0 variation years, SIP tends to minimise the variation by dividing it into two-year periods, as happened in the years 1865-66.

Table 3. Life expectancy at birth (IP, IPD, SIP ten-year mean evaluations), Sardinia 1862-1900

Periods	e_0 IP	e_0 IPD	e_0 SIP	e_0IPD/e_0IP	e_0SIP/e_0IP
1862-1870	30.5	30.3	29.5	0.994	0.966
1871-1880	30.7	30.9	30.8	1.008	1.003
1881-1890	36.2	36.1	36.2	0.997	1.000
1891-1900	38.5	38.3	38.8	0.997	1.007

We think it is important to point out (see Table 4) that comparisons which can be done (for the years 1871-72 e 1881-82) among e_0 values obtained by life tables and those estimated by the various procedures, are very encouraging: the estimates produced by IP, IPD, SIP are very close to the results obtained according to the actual knowledge of distribution by age of those living and those dead.

If we move on to analyse the obtained results (Figure 2 in Appendix A and Table 5) by means of the three procedures for the Gross Reproduction

[10] Due to the classification of deaths in classes 0 and $1-\omega$, the results between IP and IPD could be different if, for example, circumscribed epidemic would have occurred in children during their first year.

Table 4. Comparison between e_0 values (calculated in census years and estimated by IP, IPD, SIP)

Years	e_0 (life table)	e_0 IP	e_0IPD	e_0 SIP
1871-1872	28.67	27.75	28.10	28.61
1881-1882	37.61	37.45	37.35	37.67

Rate (GRR), a certain correspondence between the estimates is found. The three series point out the same trend, the IP value is generally in an intermediate position between the other two. When considering the yearly data, the differences between SIP and IPD seldom exceed 5 percent, and on average, they are definitely under this threshold. It is probably useful to point out that, unlike the stochastic version, the other procedures do not employ information about the initial age structure of the female population. This is required as input by SIP even when (as with this projection covering the period 1862-1901) only the estimates considering both sexes are under study. Thus the Stochastic Inverse Projection, with its peculiar characteristics, is particularly reliable in the fertility estimates.

Table 5. Gross reproduction rate (IP, IPD, SIP ten-year mean evaluations), Sardinia 1862-1900

Periods	GRR IP	GRR IPD	GRR SIP	GRR IPD/GRR IP	GRR SIP/GRR IP
1862-1870	2.31	2.30	2.37	0.997	1.027
1871-1880	2.41	2.44	2.38	1.012	0.988
1881-1890	2.48	2.52	2.43	1.017	0.978
1891-1900	2.39	2.36	2.30	0.985	0.960

Moving now to the consideration (Table 6) of the final age structure (1901) obtained by means of the three procedures, it can be said that these produce relatively close values, and that they are also in accordance with the corresponding distribution obtained by census data.

3 Results for the 1901-1921 Period

The aim of projections concerning the 1901-21 period is mainly to compare results of the three procedures when heavy demographic disturbances occur. In the presence of non-negligible migratory flows, the projections have been limited to women, who are certainly less involved in great migrations than

Table 6. Comparison between census age structure and estimated (IP, IPD, SIP) age structure, 1901

	0-14	15-59	60+	Total
census (1901)	32.9	58.7	8.4	100.0
IP	33.3	57.9	8.8	100.0
IPD	34.2	57.7	8.1	100.0
SIP	32.7	57.9	9.4	100.0

men. By observing Figures 3 and 4 (Appendix A) it can be clearly noted that, after a period of remarkable stability, with a minimum increase in survival together with a small fertility decrease, starting from 1916, the war determines a sizeable, progressive reduction of the survival level and a decrease in fertility levels. As far as mortality is concerned, it is the Spanish influenza epidemic occurring in 1918 which provoked the real crisis, with a considerable increase in deaths of nearly 70 percent compared with the previous year (see the data for females in Appendix C).

Let us now observe, in sequence, the values present in Table 7 (with average decennial values of life expectancy at birth), then in Table 8 (with the gross reproduction rate values) and, at last, in Table 9 (a comparison of different final age structures obtained by means of the three procedures with that obtained using the 1921 census). Basically, the results are as previously stated for the preceding period: the main trends of the demographic system seem to be similarly and effectively described by means of the three procedures.

Table 7. Life expectancy at birth (IP, IPD, SIP ten-year mean evaluations), Sardinia 1901-1920

Periods	e_0 IP	e_0 IPD	e_0 SIP	e_0IPD/e_0IP	e_0SIP/e_0IP
1901-1910	43.1	43.8	43.8	1.015	1.015
1911-1920	41.1	41.7	41.0	1.014	0.998

But it is necessary to carefully analyse e_0 annual values obtained by SIP for the years 1917 and 1918. In this case, SIP has an advantaged position when compared to IPD and IP. In fact, for the year 1918, SIP accounts for a life table correctly describing the crisis incidence on the different age

Table 8. Gross reproduction rate (IP, IPD, SIP ten-year mean evaluations), Sardinia 1901-1920

Periods	GRR IP	GRR IPD	GRR SIP	GRR IPD/GRR IP	GRR SIP/GRR IP
1901-1910	2.21	2.22	2.16	1.001	0.974
1911-1920	1.96	1.97	1.91	1.005	0.976

Table 9. Comparison between census age structure and estimated (IP, IPD, SIP) age structure, 1921

	0-14	15-59	60+	Total
census (1921)	31.1	58.4	10.5	100.0
IP	31.7	59.7	8.6	100.0
IPD	33.0	59.4	7.6	100.0
SIP	31.0	58.8	10.2	100.0

classes[11], IPD assumes more limited information (deaths distribution in two different age classes), while no additional information is given to IP[12].

Compared to the estimates produced by IP and IPD, the Stochastic Inverse Projection result (as already seen for some years of the first period) noticeably minimises the 1918 crisis (see Appendix A, Figure 3 in particular), redistributing the level fall of e_0 between 1917 and 1918. On the other hand, the data contained in Appendix C prove that the crisis has actually been limited to 1918. The absolute number of female deaths was approximately 9,600 in 1916, increasing to 10,994 in 1917, and reaching 18,421 in 1918 before dropping to 8,978 in 1919. It is common knowledge that the procedure conceived by Ronald Lee (as well as the one suggested by Alessandro Rosina) makes deaths divided by age (according to the assigned mortality model) by calculating "perspective" probabilities of dying (by completed durations), which are strictly included in each year of the calendar. Even if the procedure is not presented in detail, it is clear that the Stochastic Inverse Projection calculates "classic" probabilities of dying (between one birthday and the next, therefore in two different years). Appendix C contains those values regarding infant mortality as well as life expectancy at birth produced by means of the three

[11] The criteria adopted to the realisation of a specific mortality model for the year 1918 is shortly illustrated in the first paragraph (see note 9 in details).

[12] By using the standard Inverse Projection version (IP) it would be possible to assign a different mortality model for a single crisis year. The problem (which is also common for SIP) is that, rarely, for the past populations, it is possible to build up a life table describing the death risk distribution by age in a specific year.

procedures[13]. Without being too specific, it is clear that in the calculation of the e_0 value regarding the year 1917, SIP uses deaths which occurred in the years 1917-18, and for the calculation concerning the year 1918, on the other hand, it uses deaths which occurred in the years 1918-19, diminishing the impact of the Spanish influenza epidemic and redistributing its effect over a two-year period.

The comparison between IP and IPD for the year 1918 is interesting (see again data included in Appendix C). The two-year difference for the e_0 level (26.6 for IP and 28.5 for IPD) seems to be due to the fact that IP, by making a redistribution per age according to a standard model, over estimates infant mortality in this case. In fact, q_N values together with those of q_1 result respectively in 0.1451 and 0.1690, when the q_0 value, calculated on real data, results to be 0.1855[14]. In this case, supplying additional information to IPD (the number of deaths at 0 age separated from that of $1-\omega$ deaths) enables the production of a more precise estimate about the mortality incidence in the year of the Spanish influenza epidemic.

4 Final Remarks

Three different procedures, which recreate the demographic trend without complete information, were compared. The application of these procedures in a context where more information than is actually needed was available, has enabled us to test and verify how well the three procedures adapted to the real changes in the demographic trend. Since the aim is to point out the basic trends of fertility and mortality, though it can also be useful to employ the procedures in the annual versions for making certain specific testing, it will be especially important to highlight the e_0 and GRR smoothing values. In this sense, we can say that the most important problem for the historical demographer lies not only in the procedure's choice but also in the *quality* and *completeness* of the data and, secondly, in the choice of the models (considering also the choice of the initial population divided by age and sex - in the SIP case - since these data were rarely available in the past). We are not debating the importance of the innovative contributions made by IPD and SIP. It is clear that in the (unfortunately extremely rare) cases of past populations when we have had at our disposal some information on the deaths

[13] The first probability of dying calculated by IP and IPD is to die in the birth year, commonly indicated with the q_N symbol, so its value is quite inferior to the corresponding q_0.

[14] The latter value is obtained by relating deaths at age 0 in the year 1918 to a weighted mean of people born in 1917 and 1918. Globally, both q_N and q_1 calculated by IP deal (together) with an area of the "Lexis scheme" which is slightly wider (with respect to q_0 calculated on real data, due to the addition of deaths at age 1 deriving from those born in 1917), but not as wide to justify such a marked difference.

divided by age, the differentiated version (IPD) has more explanatory power than the original (IP) Inverse Projection version, as demonstrated by Alessandro Rosina in the study on the transition in the Venetian area[15]. Similarly, the contribution given by the Stochastic Inverse Projection, which goes beyond the deterministic logic and shows various possible evolution mechanisms of the populations analysed, seems important to us since it helps researchers using these methods to abandon misleading certitudes and to, in short, consider results only as "potential background" of the demographic system.

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[15] See [11, 15].

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Appendix A

Sardinia - Annual demographic indicators (expectation of life at birth and gross reproduction rate) estimated by Inverse Projection (IP), Differentiated Inverse Projection (IPD), Stochastic Inverse Projection (SIP)

Table 10. Expectation of life at birth (e_0)

Year	Males and Females			Year	Females		
	IP	IPD	SIP		IP	IPD	SIP
1862	32.3		28.7	1901	43.2	43.9	43.8
1863	26.1	25.6	26.8	1902	43.3	44.0	43.8
1864	28.4	28.1	28.2	1903	43.7	44.6	43.6
1865	28.8	28.8	31.6	1904	42.5	43.1	41.5
1866	37.5	38.1	34.3	1905	40.5	41.0	42.3
1867	32.6	32.6	29.3	1906	44.3	44.9	44.7
1868	26.4	26.6	28.1	1907	43.6	44.4	45.1
1869	30.9	31.3	29.9	1908	44.3	44.8	44.1
1870	29.6	29.8	28.4	1909	41.7	42.2	43.8
1871	27.0	27.5	27.7	1910	44.1	44.8	44.8
1872	28.4	28.6	29.5	1911	43.3	43.9	44.6
1873	31.1	31.4	31.5	1912	45.3	45.7	45.7
1874	32.3	32.5	31.3	1913	44.0	44.3	45.5
1875	30.4	30.7	30.5	1914	45.4	45.9	45.4
1876	30.5	30.9	31.1	1915	43.3	43.6	43.4
1877	31.9	32.2	32.0	1916	42.3	42.6	40.7
1878	32.2	32.7	32.1	1917	38.9	39.5	30.0
1879	31.8	32.2	30.6	1918	26.6	28.5	29.6
1880	29.1	29.7	31.4	1919	40.9	41.7	41.5
1881	34.2	34.3	36.5	1920	40.9	41.0	43.8
1882	39.5	39.2	38.9	1921	44.1	43.9	
1883	38.0	37.9	37.0				
1884	35.3	35.3	34.8				
1885	33.4	33.3	34.7				
1886	35.1	35.2	36.1				
1887	36.5	36.5	36.7				
1888	36.0	36.1	36.3				
1889	35.6	35.6	35.5				
1890	34.2	34.5	35.8				
1891	36.4	36.4	37.0				
1892	36.7	36.8	38.0				
1893	38.5	38.3	37.7				
1894	35.8	35.9	37.2				
1895	37.1	37.3	39.3				
1896	40.9	40.8	40.8				

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Table 10. *(continued)*

Year	Males and Females		
	IP	IPD	SIP
1897	39.7	39.6	40.4
1898	40.0	39.9	40.0
1899	38.4	38.2	39.0
1900	37.8	37.7	38.1

Table 11. Gross Reproduction Rate (GRR)

Year	Males and Females			Year	Females		
	IP	IPD	SIP		IP	IPD	SIP
1862	2.24		2.29	1901	2.14	2.14	2.13
1863	2.24	2.25	2.30	1902	2.28	2.28	2.21
1864	2.30	2.25	2.37	1903	2.18	2.19	2.14
1865	2.34	2.31	2.39	1904	2.24	2.24	2.21
1866	2.39	2.34	2.45	1905	2.16	2.16	2.12
1867	2.35	2.39	2.41	1906	2.21	2.21	2.15
1868	2.14	2.35	2.20	1907	2.23	2.23	2.14
1869	2.36	2.14	2.45	1908	2.34	2.34	2.25
1870	2.39	2.35	2.44	1909	2.14	2.14	2.05
1871	2.29	2.38	2.28	1910	2.21	2.22	2.16
1872	2.46	2.29	2.42	1911	2.12	2.13	2.11
1873	2.44	2.46	2.39	1912	2.08	2.09	2.07
1874	2.45	2.45	2.43	1913	2.15	2.16	2.10
1875	2.46	2.47	2.42	1914	2.03	2.03	1.98
1876	2.46	2.49	2.42	1915	2.15	2.16	2.09
1877	2.46	2.49	2.43	1916	1.91	1.92	1.84
1878	2.35	2.50	2.33	1917	1.69	1.70	1.60
1879	2.42	2.40	2.42	1918	1.45	1.46	1.40
1880	2.33	2.48	2.29	1919	1.80	1.82	1.74
1881	2.44	2.39	2.42	1920	2.18	2.19	2.16
1882	2.51	2.50	2.46	1921	1.98	2.00	
1883	2.44	2.57	2.39				
1884	2.52	2.50	2.48				
1885	2.56	2.58	2.50				
1886	2.45	2.61	2.39				
1887	2.53	2.49	2.47				
1888	2.46	2.57	2.40				
1889	2.53	2.48	2.46				
1890	2.37	2.55	2.29				
1891	2.47	2.37	2.40				
1892	2.47	2.46	2.37				
1893	2.54	2.45	2.45				
1894	2.40	2.51	2.29				

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Table 11. *(continued)*

Year	Males and Females		
	IP	IPD	SIP
1895	2.35	2.36	2.25
1896	2.42	2.30	2.31
1897	2.38	2.35	2.26
1898	2.32	2.30	2.22
1899	2.30	2.24	2.20
1900	2.26	2.21	2.20

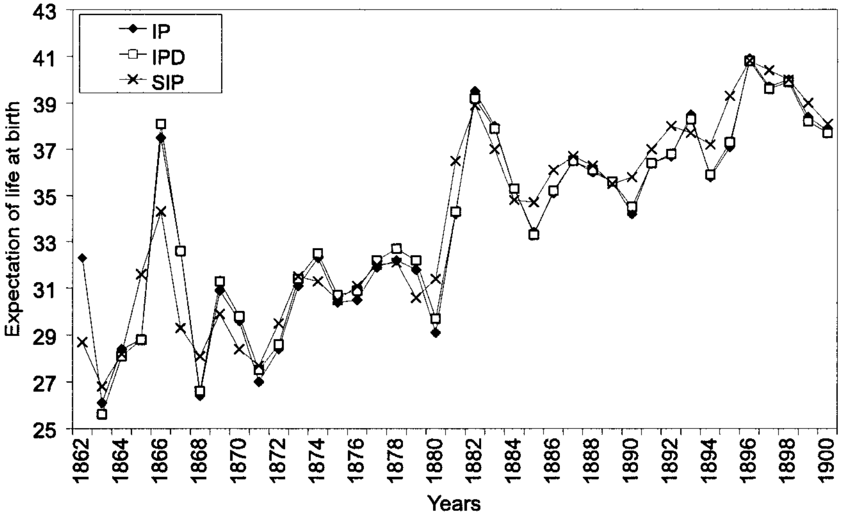

Figure 1. Comparison among e_0 (males+females) values (IP, IPD, SIP), Sardinia 1862-1900

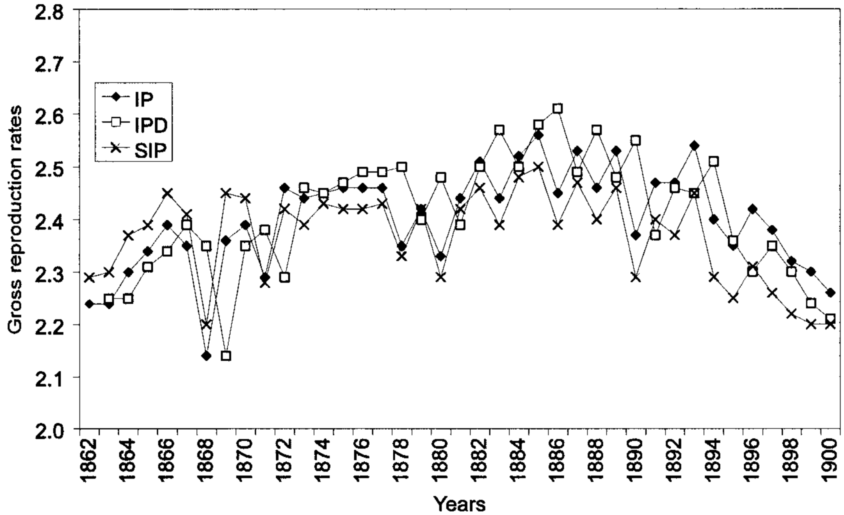

Figure 2. Comparison among female GRR values (IP, IPD, SIP), Sardinia 1862-1900

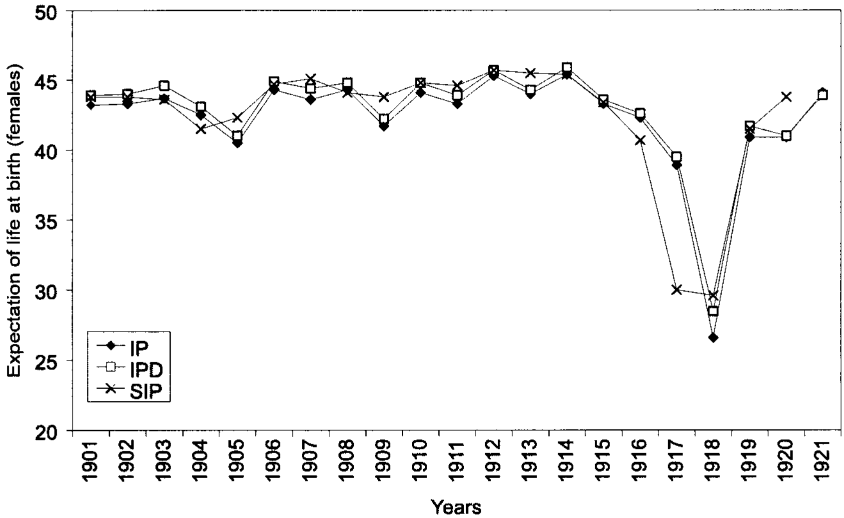

Figure 3. Comparison among e_0 (females) values (IP, IPD, SIP), Sardinia 1901-1921

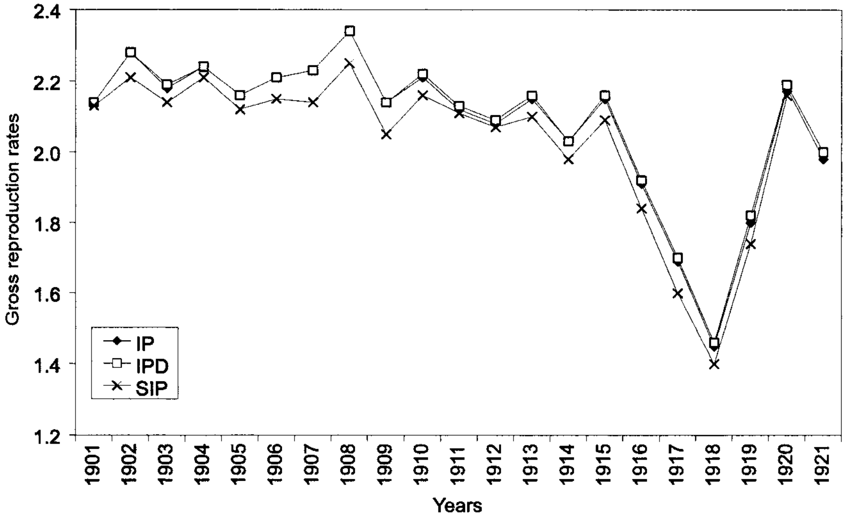

Figure 4. Comparison among female GRR values (IP, IPD, SIP), Sardinia 1901-1921

Appendix B

Table 12. Age distribution of deaths 1918 and 1921 (base 100000)

Age	1921	1918
0	12417	10058
1	5407	5383
2	2545	3329
3	1368	2236
4	834	1243
5	692	1123
6	596	1039
7	520	878
8	465	904
9	428	835
10	355	801
11	328	654
12	304	572
13	306	628
14	318	591
15	331	642
16	362	703
17	386	750
18	420	816
19	456	887
20	494	966
21	534	1045
22	565	1106
23	583	1141
24	579	1133
25	575	989
26	571	980
27	567	975
28	574	987
29	583	1001
30	579	839
31	576	834
32	583	844
33	591	855
34	587	849
35	594	824
36	601	834
37	595	826
38	586	813
39	563	781
40	542	709

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Table 12. *(continued)*

Age	1921	1918
41	520	680
42	500	654
43	508	663
44	507	662
45	516	611
46	515	609
47	526	622
48	549	650
49	558	661
50	569	590
51	593	615
52	617	640
53	653	677
54	673	698
55	709	611
56	746	644
57	780	672
58	822	708
59	874	754
60	913	779
61	962	820
62	1023	872
63	1111	947
64	1208	1030
65	1325	1072
66	1444	1169
67	1579	1278
68	1703	1379
69	1786	1445
70	1857	1333
71	1993	1431
72	2111	1516
73	2208	1585
74	2277	1634
75	2313	1692
76	2314	1693
77	2277	1666
78	2203	1612
79	2094	1532
80	1954	1166
81	1790	1068
82	1608	960
83	1417	845
84	1223	730

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Table 12. *(continued)*

Age	1921	1918
85	1035	655
86	858	543
87	696	440
88	554	350
89	431	273
90	329	196
91	246	147
92	180	107
93	129	77
94	90	54
95	62	32
96	42	22
97	28	14
98	18	9
99	11	6
Total	100000	100000

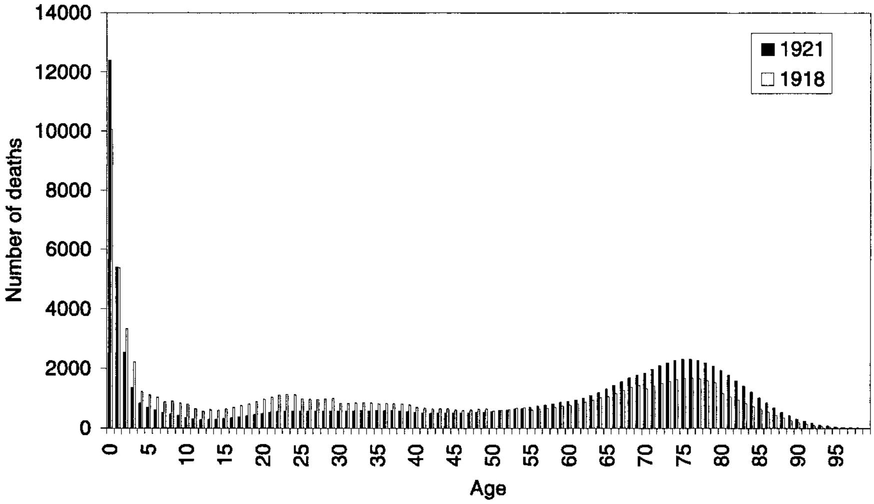

Figure 5. Age distribution of deaths 1921 and 1918 (base 100000), Sardinia

Appendix C

Table 13. Infant mortality and life expectancy at birth estimated by SIP (probabilities between exact ages) and IP, IPD (probabilities between completed ages)

Years	Live births	Tot deaths	Deaths 0-1	q_0	q_N IP	q_1 IP	q_0 SIP	q_1 SIP	q_N IPD	q_1 IPD
Period 1875-78 (Males and Females)										
1874	24758									
1875	24914	20818	4589	0.1846	0.1350	0.1451	0.1948	0.1273	0.1012	0.0934
1876	24933	20886	4230	0.1697	0.1346	0.1446	0.1923	0.1240	0.0949	0.0838
1877	24976	20054	4374	0.1752	0.1309	0.1388	0.1862	0.1200	0.0969	0.0868
1878	23911	19885	4222	0.1740	0.1301	0.1376	0.1840	0.1200	0.0943	0.0829
Period 1916-19 (Females)										
1915	13548	9230	1659							
1916	12075	9627	1641	0.1306	0.0916	0.0903	0.1448	0.0778	0.0719	0.0616
1917	10689	10994	1642	0.1472	0.1003	0.1030	0.1693	0.0990	0.0787	0.0716
1918	9188	18421	1797	0.1855	0.1456	0.1690	0.1694	0.0994	0.0945	0.0945
1919	11448	8978	1225	0.1145	0.0950	0.0952	0.1462	0.0706	0.0685	0.0567

Males and Females

	e_0		
Years	IP	IPD	SIP
1875	30.4	30.7	30.5
1876	30.5	30.9	31.1
1877	31.9	32.2	32.0
1878	32.2	32.7	32.1

Females

	e_0		
Years	IP	IPD	SIP
1916	42.3	42.6	40.7
1917	38.9	39.5	30.0
1918	26.6	28.5	29.6
1919	40.9	41.7	41.5