IV. Conclusions

Afterthoughts on the mortality tempo effect

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Summary. The preceding chapters in this volume provide a broad ranging and stimulating analysis of our claim that conventional estimates of period life expectancy may be distorted by a mortality tempo effect. Much new insight into the process of mortality change and its measurement has been gained, but there is no clear consensus on the existence, nature and size of the tempo effect. Views from different contributors range widely from strongly supportive to dismissive.

The purpose of this note is to comment briefly on the main question raised about our analysis of the mortality tempo effect: Is our tempo adjusted life expectancy a current measure of mortality conditions as we (and Vaupel in this volume p. 93 and Guillot in this volume) believe or a measure of the past as suggested by Rodriguez (in this volume) and Wachter (in this volume)?

1 Do tempo adjusted period longevity measures reflect current mortality conditions?

Conventional analyses of levels and trends in period mortality indicators such as life expectancy at birth are based on the assumption that current mortality *rates* measure current mortality *conditions*. Vaupel (2002, in this volume p. 93) concludes that mortality rates do not necessarily represent mortality conditions because of heterogeneity in mortality risks and/or because of delays in deaths, where "death delays" refer to the empirical tendency of survival curves to shift uniformly to the right at ages beyond young adulthood. We focus here on the role of death delays.

To clarify what is meant by "conditions" we turn to a published comment of Hajnal (1948). Discussing the relation between the marriage rate and the population age and sex structure he observes that

> ...we must know how, given the present marriage habits of the population, marriage rates would change as the age and sex structure of the population changes.

The remark is notable as an early articulation of the idea that demographic phenomena may or may not be adequately described by *rates* in the conventional sense of the word.

By "conditions" we mean what Hajnal meant by "habits:" an idea about how the phenomena in question "works" that may be represented by a mathematical model. The model will express the phenomena in question in terms of one or more parameters that describe the state of the system. If the values of these parameters cannot be directly observed, we will search for observable quantities that provide information on (and preferably determine) the parameter values.

The distinction between rates and conditions implies two potentially different period longevity measures defined as follows:

$e_0(t)$ = the mean age at death implied by current mortality *rates*
$M(t)$ = the mean age at death implied by current mortality *conditions*

We will refer to $e_0(t)$ as the conventional or unadjusted life expectancy and to $M(t)$ as the adjusted life expectancy. The interpretation of these period measures and any distortions in them depends on how the underlying process is modeled, as we discuss below.

1.1 The conventional "rates" perspective

The conventional "rates" perspective on the measurement of longevity holds that observed mortality rates (hazards or rates of the first kind) are the appropriate measures of period mortality and $e_0(t)$, calculated from these rates, is the most appropriate measure of period longevity. All other mortality variables, including $M(t)$, are derived from these rates. Expectation of life at birth, for example, is calculated as

$$e_0(t) = \int_0^\infty \exp\left(-\int_0^a \mu(x,t)\,dx\right)\,da. \tag{1}$$

where $\mu(x,t)$ denotes the force of mortality at age x and time t.

As Wachter (in this volume) points out, our estimate of $M(t)$, obtained with equation (3) below, is related to the conventional period life expectancy as

$$M(t) \approx \int_{-\infty}^t w_t(x) e_0(x)\,dx \tag{2}$$

thus making $M(t)$ a weighted average of $e_0(t)$. (We follow Wachter's simplifying assumptions and use his notation)

The conception of "how mortality works" in this context, how the underlying process is modeled, is rarely made explicit, but a plausible underlying model is that of organisms exposed to shocks and stresses imposed by their environment that may result in immediate death. This being the case, the numbers of deaths will be linear function of the number of persons exposed to risk, which function is fully specified by the ratio of deaths to persons at risk. Mortality *rates* are in this case a faithful representation mortality *conditions*.

1.2 The "conditions" perspective

In what we call the "conditions" perspective, adult mortality in high life expectancy populations "works" differently. Individuals die when their allotment of life has been exhausted. An allotment can increase or decrease over time as mortality conditions in successive periods vary during the individual's life. In this perspective the most appropriate measure of period longevity is $M(t)$, with all other mortality variables, including $e_0(t)$, derived from these fundamental conditions.

$M(t)$ is a period indicator of current mortality conditions and is defined as the life expectancy of the cohort born in year t if no further changes in conditions occur after time t (see Vaupel in this volume p. 93 and Guillot in this volume for similar definitions).

As shown by Vaupel (in this volume p. 93) current conditions by age can be estimated as $\frac{\mu(x,t)}{1-\delta(t)}$ and the adjusted period life expectancy of life is given by

$$M(t) = \int_0^\infty \exp\left(-\int_0^a \frac{\mu(x,t)}{1-\delta(t)}\, dx\right) da \qquad (3)$$

where $\delta(t)$ equals the increment to life at time t, i.e., the addition made to the life lines of everyone alive at time t as mortality declines.

Equation (3) is the same as the one provided by Bongaarts and Feeney (in this volume p. 11 and p. 29), who estimate $\delta(t)$ as the rate of change in the adjusted life expectancy, $\delta(t) = \frac{dM(t)}{dt}$, and present methods for the estimation of $\delta(t)$. Note that nothing on the right side of equation (3) depends on the past: $\mu(x,t)$ is the current force of mortality and $\delta(t)$ equals the delay in the timing of future deaths caused by changes in conditions at time t.Vaupel (in this volume p. 93) calls $M(t)$ the "true" life expectancy at birth. Guillot (in this volume) concludes that $M(t)$ can be interpreted as an indicator reflecting current mortality conditions under specific assumptions.

In the mortality conditions perspective, the conventional period life expectancy is considered distorted and it is determined by $M(t)$ as follows

$$e_0(t) = M(t) + g(t)\frac{dM(t)}{dt} \qquad (4)$$

This equation is obtained by rearranging equation (7) in Wachter (in this volume); Bongaarts and Feeney (2002) and Guillot (2003, in this volume) provide similar equations. (For Gompertz mortality with a fixed slope, $g(t) \approx \beta^{-1}$.)

Equation (4) shows that conventional period life expectancy differs from the adjusted life expectancy $M(t)$ by an amount that depends on the rate at which $M(t)$ is changing. This means that $e_0(t)$ will be a distorted measure of period longevity implied by current mortality conditions. The difference between $e_0(t)$ and $M(t)$ is the mortality tempo effect.

It is important to note that (2) and (4) are both expressions relating $e_0(t)$ and $M(t)$. In fact, (2) is the solution to differential equation (4), which means that substitution of (2) in (4) yields an identity. The difference between these equations is that $M(t)$ is the independent variable in (4), whereas $e_0(t)$ is the independent variable in (2).

The chapters by Rodriguez, and Wachter focus on the conventional rates perspective. In this perspective, current rates and the $e_0(t)$ calculated from them are not distorted and $M(t)$ depends on past rates. Vaupel (2002) and Bongaarts and Feeney (2002, in this volume p. 11 and p. 29) focus on the perspective in which individuals die when their allotment of life has been exhausted. In this perspective, $M(t)$ is independent of the past force of mortality, as shown by (3), and $e_0(t)$ is distorted. Guillot (in this volume) provides descriptions and insightful comments on these two perspectives.

The preceding discussion contrasts two quite different perspectives, based on two different models for the process of mortality, but it leaves open the question as to which model is the better representation of the reality of human mortality. A full discussion of this issue is beyond the scope of this note but we believe that the rates perspective is largely correct for causes of deaths that occur more or less at random (e.g. deaths from infection, accidents and violence, which predominate in childhood and among young adults). In contrast the conditions perspective is correct for mortality at older ages when deaths do not occur randomly but are instead the result of senescence. Senescence refers to the slow deterioration of cellular and physiological processes which precedes deaths from degenerative diseases, mostly above about age 30. This is why Bongaarts and Feeney (2002, in this volume p. 11 and p. 29) restrict their analysis of the mortality tempo effect to ages above 30.

1.3 An illustration

To clarify the distinction between the rates and conditions perspectives we will now present a brief analysis of these contrasting approaches in a model population. In this population every newborn receives a ticket with a predetermined age at death (a random variable). Let $T(t)$ denote the average age on the tickets issued in year t, but the age on any person's ticket can be changed at any time during the person's life, for example if the person lives through a year in which medical or public health discoveries occur. Innovations in year t (e.g. new drugs, surgical techniques) in medicine or public health raise everyone's life expectancy (the ticket value) provided the innovations remain effective over time.

Let $r(t)$ denote the increment to the age on the ticket made in year t. Suppose that the increment $r(t)$ may vary from year to year, but it is the same for all individuals alive at time t. This implies that the increment to the value of the ticket does not depend on the age of the person holding it.

To illustrate, suppose that the average value of these tickets has been constant and equal to $T(0)$ until year 0 (i.e. $r(t) = 0$ for $t < 0$), and that mortality improvements occur after $t = 0$. The average value of the ticket given to a newborn in year t then equals $T(0)$ plus the sum of all improvements between years 0 and t so that in continuous time

$$T(t) = T(0) + \int_0^t r(x)\, dx \tag{5}$$

and

$$r(t) = \frac{dT(t)}{dt} \tag{6}$$

In this model population the above equations apply and it can be shown that

a. the adjusted life expectancy, which measures current conditions, equals $T(t)$, because $T(t)$ equals the mean age at death of the cohort born at time t if no further improvements in mortality conditions occur in the future:

$$M(t) = T(t) \tag{7}$$

b. the conventional unadjusted life expectancy differs from the ticket value and hence from $M(t)$, and the difference depends on the rate of improvement in mortality conditions. This follows from equation (4) :

$$e_0(t) > M(t) \quad \text{for } r(t) > 0 \tag{8}$$

and

$$e_0(t) < M(t) \quad \text{for } r(t) < 0$$

That is, when mortality conditions are improving the conventional life expectancy derived from rates exceeds the ticket value $M(t)$ and the reverse is true when the mortality conditions are deteriorating. The difference between these measures is the mortality tempo effect which varies with the value of $r(t)$ but is independent of the past. (If mortality follows a Gompertz the tempo effect equals approximately $\frac{r(t)}{\beta}$). The reason for the existence of the tempo effect is the thinning out of events in any year in which $r(t)$ is positive. As conditions improve in year t deaths that would have occurred in the year t without the improvement are postponed to some future year thus reducing the density of deaths in year t. This postponement and hence the thinning out of events continues as long as conditions keep improving (e.g. with constant non zero $r(t)$, the values of $e_0(t)$ and $M(t)$ will differ but they will change over time at the same pace).

As shown by Bongaarts and Feeney (2002, in this volume p. 11 and p. 29) the distortion caused by this thinning can be removed by making an adjustment which divides the observed but distorted force of mortality by $(1 - r(t))$. Using this adjusted force of mortality in a conventional life table yields $M(t)$ which equals $T(t)$. Vaupel (in this volume p. 93) has a very similar view of this process (in his chapter he uses $\delta(t)$ for $r(t)$)

In sum, as noted by Vaupel (2003), life expectancy under current conditions does not equal life expectancy under current rates. The conventional period life expectancy is of course a summary measure of current rates ($e_0(t)$ in fact equals the inverse of the weighted average of age specific mortality rates), but when mortality conditions are changing $e_0(t)$ does not measure these conditions accurately. Under the specified simplifying assumptions, our adjusted life expectancy measures the life expectancy implied by current mortality conditions and it is therefore not a measure of past mortality conditions.

2 Conclusion

The calculation of period life expectancy from hazard rates with conventional mortality life tables originated more than two centuries ago, when infectious diseases were the primary causes of death and life expectancy at birth in European countries was less than half of current levels. The contemporary "model" for human mortality seems never to have been made explicit, but it evidently embodied the idea of people being "struck down" by events in the environment. This model is far less relevant today than it was two centuries ago, but we are so accustomed to the rates perspective on which the period life table is based that we tend to accept it without question.

Our research into tempo effects has lead us to a thorough reconsideration of the fundamentals of mortality measurement. We take it for granted that measurement is based on some understanding of the process that generates the observed phenomena, deaths in this case. If the nature of the phenomena changes, as it has with respect to mortality, it is appropriate to reconsider whether existing measurements are still appropriate. We agree with Vaupel that, with respect to measurement of mortality, this is often not the case.

This brief note documents the mathematical relationships between longevity measures derived from the rates and conditions perspectives. We believe that the assumptions underlying the latter are applicable to senescent mortality which dominates in contemporary low mortality countries. In the conditions perspective the conventional period life expectancy gives a distorted estimate of the life expectancy implied by current mortality conditions. This tempo distortion is positive when mortality conditions are improving and negative when they are deteriorating. Most countries are currently experiencing improvements, and their conventionally calculated period life expectancies therefore have an upward distortion.

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