
Turbulence in lifetables: Demonstration by four simple examples

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Summary. To understand why mortality change can distort calculations of death rates and life expectancy, it is informative to consider some examples that are as simple as possible. This short chapter presents four such illustrations. They show how lifesaving can roil lifetable statistics.

1 Introduction

Demographic change can distort demographic rates. In particular, mortality change can distort period death rates, life expectancy and other lifetable statistics. This fact is considered from various perspectives in earlier chapters of this monograph, including my chapter on “Lifesaving, lifetimes and lifetables”. Some demographers, however, still do not accept the surprising and uncomfortable reality. The main reason seems to be that published explanations, including those in this book, are difficult to comprehend. My chapter and my earlier article on “Life expectancy at current rates vs. current conditions” are hard to read, in large part because they were hard to write. John Bongaarts and Griff Feeney started to correspond with me about tempo effects on mortality in early 2001. It took me hundreds of hours of thinking, spread out over several years, before I finally understood the issues sufficiently well to feel confident about my comprehension and it is only recently that I have been able to explain to students why demographic change roils period rates.

As with other puzzling concepts that seem counterintuitive given our education (but eventually become intuitive when we re-educate ourselves), the best entre to comprehending mortality turbulence is to consider some examples that are as simple as possible. This short chapter presents four such illustrations.

2 Saving infant lives for one year

Consider the special case of a closed population with a constant mortality regime that has prevailed for many years. Suppose there are only four ages, ages 0, 1, 2 and 3. Each year exactly 100 individuals are born, all on January 1st. All deaths occur on July 1st, halfway through the year. The last survivors die at exact age 3.5. The first three columns of Table 1 provide statistics about this prevailing regime. Exactly 40 individuals die at age 0, 20 at age 1, 20 at age 2 and 20 at age 3. Life expectancy at birth is 1.7 years. The statistics for each period are identical to the corresponding statistics for each birth cohort. To highlight the fact that individuals age along diagonals, the population sizes and death counts for the cohorts born in years 1 and 4 are shaded light grey.

Table 1. Population statistics at different ages and times when 30 lives are saved at age 0 and extended by one year.

Age	Statistic	Year						
		1	2	3	4	5	6	7
0	e	<i>1.7</i>	<i>1.7</i>	<i>1.7</i>	2.3	2	2	2
	P	100	100	100	100	100	100	100
	D	40	40	40	10	10	10	10
	q	0.4	0.4	0.4	0.1	0.1	0.1	0.1
1	e	<i>1.5</i>	<i>1.5</i>	<i>1.5</i>	1.5	1.17	1.17	1.17
	P	60	60	60	60	90	90	90
	D	20	20	20	20	50	50	50
	q	0.33	0.33	0.33	0.33	0.556	0.556	0.556
2	e	<i>1</i>	<i>1</i>	<i>1</i>	1	1	1	1
	P	40	40	40	40	40	40	40
	D	20	20	20	20	20	20	20
	q	0.5	0.5	0.5	0.5	0.5	0.5	0.5
3	e	<i>0.5</i>	<i>0.5</i>	<i>0.5</i>	0.5	0.5	0.5	0.5
	P	20	20	20	20	20	20	20
	D	20	20	20	20	20	20	20
	q	1	1	1	1	1	1	1
Total deaths		100	100	100	70	100	100	100

Note: The new mortality regime starts in year 4. The letter e denotes remaining life expectancy, P population size, D number of deaths and q probability of death. The values in italics pertain to times prior to the mortality shift. The values in bold are discordant with subsequent values prevailing after the shift. Population sizes and death counts for the cohorts born in years 1 and 4 are shaded grey.

At the start of year 4, a new mortality regime replaces the old one. The number of deaths at age zero is reduced from 40 to 10. The 30 individuals whose lives are saved die at age one: their lives are extended by exactly one year. This is the only change in the mortality regime—but it results in changes in various statistics.

Note that the number of deaths in year 4 is 70 rather than 100. This reflects the fact that 30 lives were saved. These individuals, however, die in year 5, so in year 5 the number of deaths returns to the stationary level of 100 per year. As in year 4, the lives of 30 infants are saved in year 5 and in subsequent years, but starting in year 5 the resuscitated die, adding 30 deaths that balance the 30 lives saved.

Examine the number of age-specific deaths each year. Note that at age 0 this number falls from 40 to 10 in year 4, but that it rises at age 1 from 20 to 50 in year 5.

Consider the age-specific probabilities of death. Note that $q(0)$ drops from .4 to .1 in year 4 and afterwards and that $q(1)$ rises and does so starting in year 5. This is a very simple example of the “delayed gerontological failure of pediatric success”: saving lives at younger ages can result in higher death rates at older ages in later years.

In the 4th year, 60 individuals are alive at the start of age 1. They are the survivors of the 100 individuals born in the 3rd year. The new mortality regime does not affect them: as in earlier years 20 of them die and the probability of death remains 0.33. Because remaining life expectancy at age 2 stays at 1.0, remaining life expectancy at age 1 stays at a level of 1.5. This is readily verified by using the formula:

$$e(1) = 0.5q(1) + (1 + e(2))(1 - q(1)) .$$

The formula is true because those who die in the year die halfway through the year, adding half a year to life expectancy, whereas those who survive get a year of life that year plus remaining life expectancy at age 2.

A similar formula can be used to calculate life expectancy at birth:

$$e(0) = 0.5q(0) + (1 + e(1))(1 - q(0)) .$$

The formula yields a life expectancy at birth of 2.3, some 0.6 years higher than the previous level of 1.7.

In year 5 and thereafter life expectancy at birth is calculated not as 2.3 but as 2.0. The value of 2.0 is the correct value for the new regime: 30 lives—30% of the number of births—were saved for one year, adding $0.3 \times 1 = 0.3$ to the previous life expectancy of 1.7. The underlying mortality regime in year 4 is the same as in later years, but the calculated $q(1)$'s are different. The reason is that at age 1 in year 4 there were 20 deaths out of a population of 60 but in subsequent years there are 50 deaths out of a population of 90. The population size at age 1 in year 5 is 90 and the number of deaths is 50

because 30 lives were saved for one year at age 0 the year before. These saved individuals do not contribute to the death count in year 4.

This simple example is proof that mortality change can roil lifetable statistics. And the example points to the explanation. A second example helps clarify the nature of the turbulence.

3 Saving infant lives for three years

In Table 2, the mortality regime before the shift is the same as in Table 1. Now, however, the 30 lives saved at age zero are extended not by one but by three years. Note that the column of statistics for year 4 is identical for Tables 1 and 2. The same number of lives are saved—and it is this that determines the column of statistics. How long the lives are extended has no impact on year 4. Hence, as in Table 1, life expectancy at birth in year 4 in Table 2 is calculated as 2.3 years.

Lifetable statistics are also distorted in year 5. Note that the values of e , P , D and q at age 2 in year 5 are the same as they were in earlier years—even though the mortality regime changed in year 4. The underlying reason for this is that those saved in year 4 have not died yet—they first die in year 7.

As shown for years 6 and 7, life expectancy under the new regime is not 2.3 or 2.45 years but 2.6 years. Under the old regime, life expectancy was 1.7. Under the new regime, 30% of the population at age zero gains 3 years of life. Hence the new life expectancy is 1.7 plus .3 times 3, or 2.6. The erroneous values of 2.3 and 2.45 calculated in years 4 and 5 are simply a result of the fact that 30 deaths were averted in year 4 and it is not known, in years 4 and 5, when these individuals will die. In year 6 and subsequently, it is known—they will die at age 3. Hence the true life expectancy can be calculated.

4 Saving infant lives for two years on average

In Table 3, some 30 lives are also saved starting in year 4, but now 10 of them are extended by one year, 10 by two years and 10 by three years. As in Tables 1 and 2, life expectancy at birth in year 4 is computed as 2.3, but now this turns out to be the correct value. The age-specific population sizes and death counts do not reach a new equilibrium until year 5 at age 1, year 6 at age 2 and year 7 at age 3. The probabilities of death, however, that are implied by these numbers are the correct values and so are the resulting calculations of age-specific life expectancies.

The reason is as follows. In computing lifetables using standard methods, it is assumed that all persons have the same life chances. If lives are saved, then it is assumed that those saved get the same life chances as those who would have survived anyway. In Tables 1, 2, and 3, the individuals who survived to age 1 under both the old and new regimes faced probabilities of death such

Table 2. Population statistics at different ages and times when 30 lives are saved at age 0 and extended by three years.

Age	Statistic	Year						
		1	2	3	4	5	6	7
0	e	<i>1.7</i>	<i>1.7</i>	<i>1.7</i>	2.3	2.45	2.6	2.6
	P	100	<i>100</i>	<i>100</i>	100	100	100	100
	D	40	<i>40</i>	<i>40</i>	10	10	10	10
	q	<i>0.4</i>	<i>0.4</i>	<i>0.4</i>	0.1	0.1	0.1	0.1
1	e	<i>1.5</i>	<i>1.5</i>	<i>1.5</i>	1.5	1.67	1.17	1.17
	P	<i>60</i>	60	<i>60</i>	60	90	90	90
	D	<i>20</i>	20	<i>20</i>	20	20	20	20
	q	<i>0.33</i>	<i>0.33</i>	<i>0.33</i>	0.33	0.222	0.222	0.222
2	e	<i>1</i>	<i>1</i>	<i>1</i>	1	1	1.214	1.214
	P	<i>40</i>	<i>40</i>	40	40	40	70	70
	D	<i>20</i>	<i>20</i>	20	20	20	20	20
	q	<i>0.5</i>	<i>0.5</i>	<i>0.5</i>	0.5	0.5	0.286	0.286
3	e	<i>0.5</i>	<i>0.5</i>	<i>0.5</i>	0.5	0.5	0.5	0.5
	P	<i>20</i>	<i>20</i>	<i>20</i>	20	20	20	50
	D	<i>20</i>	<i>20</i>	<i>20</i>	20	20	20	50
	q	<i>1</i>	<i>1</i>	<i>1</i>	1	1	1	1
Total deaths		100	100	100	70	70	70	100

Note: The new mortality regime starts in year 4. The letter e denotes remaining life expectancy, P population size, D number of deaths and q probability of death. The values in italics pertain to times prior to the mortality shift. The values in bold are discordant with subsequent values prevailing after the shift. Population sizes and death counts for the cohorts born in years 1 and 4 are shaded grey.

that a third of them died at age 1, a third at age 2 and a third at age 3. In Table 3, the individuals whose lives were saved at age 0 got exactly the same life chances—a third of them die at age 1, a third at age 2 and a third at age 3. But in Tables 1 and 2 the resuscitated face different mortality schedules than those not saved. In Table 1 they have a 100% chance of death at age 1; in Table 3 they have a 100% chance of death at age 3.

5 Saving everyone’s life for one year

How is “turbulence in lifetables” illustrated above related to “tempo effects on mortality”? If tempo effects are broadly defined as distortions arising from

Table 3. Population statistics at different ages and times when 30 lives are saved at age 0, with 10 extended by one year, 10 by two years and 10 by three years.

Age	Statistic	Year						
		1	2	3	4	5	6	7
0	e	<i>1.7</i>	<i>1.7</i>	<i>1.7</i>	2.3	2.3	2.3	2.3
	P	100	<i>100</i>	<i>100</i>	100	100	100	100
	D	40	<i>40</i>	<i>40</i>	10	10	10	10
	q	<i>0.4</i>	<i>0.4</i>	<i>0.4</i>	0.1	0.1	0.1	0.1
1	e	<i>1.5</i>	<i>1.5</i>	<i>1.5</i>	1.5	1.5	1.5	1.5
	P	<i>60</i>	60	<i>60</i>	60	90	90	90
	D	<i>20</i>	20	<i>20</i>	20	30	30	30
	q	<i>0.33</i>	<i>0.33</i>	<i>0.33</i>	0.33	0.33	0.33	0.33
2	e	<i>1</i>	<i>1</i>	<i>1</i>	1	1	1	1
	P	<i>40</i>	<i>40</i>	40	40	40	60	60
	D	<i>20</i>	<i>20</i>	20	20	20	30	30
	q	<i>0.5</i>	<i>0.5</i>	<i>0.5</i>	0.5	0.5	0.5	0.5
3	e	<i>0.5</i>	<i>0.5</i>	<i>0.5</i>	0.5	0.5	0.5	0.5
	P	<i>20</i>	<i>20</i>	<i>20</i>	20	20	20	30
	D	<i>20</i>	<i>20</i>	<i>20</i>	20	20	20	30
	q	<i>1</i>	<i>1</i>	<i>1</i>	1	1	1	1
Total deaths		100	100	100	70	80	90	100

Note: The new mortality regime starts in year 4. The letter e denotes remaining life expectancy, P population size, D number of deaths and q probability of death. The values in italics pertain to times prior to the mortality shift. The values in bold are discordant with subsequent values prevailing after the shift. Population sizes and death counts for the cohorts born in years 1 and 4 are shaded grey.

changes over time in death rates, then the two phrases are synonymous. Bongaarts and Feeney introduced the concept of tempo effects on mortality with a more specific meaning: tempo effects result from changes in mortality that uniformly extend (or shorten) everyone's remaining lifespan by the same amount. This is analogous to their concept of tempo effects on fertility and is in keeping with the meaning of tempo in music. Bongaarts and Feeney's idea is so interesting and stimulating that it seems appropriate to use "tempo effects on mortality" to describe it. Table 4 provides a simple illustration, albeit a crude one because Bongaarts and Feeney hypothesize that tempo effects occur only at adult ages.

In year 4 everyone's life is extended by one year. Those who would have died at age 0 now die age 1 (one year later). Similarly those who would have died at age 1, 2 or 3 now die at age 2, 3 or 4—again, one year later. Because no one dies in year 4, life expectancy is infinite. Actually, however, life expectancy has been extended exactly one year—from 1.4 years to 2.4 years.

Table 4. Population statistics at different ages and times when all lives at all ages are saved and extended by one year.

Age	Statistic	Year						
		1	2	3	4	5	6	7
0	e	<i>1.7</i>	<i>1.7</i>	<i>1.7</i>	∞	2.7	2.7	2.7
	P	100	100	100	100	100	100	100
	D	40	40	40	0	0	0	0
	q	<i>0.4</i>	<i>0.4</i>	<i>0.4</i>	0	0	0	0
1	e	<i>1.5</i>	<i>1.5</i>	<i>1.5</i>	∞	1.7	1.7	1.7
	P	<i>60</i>	60	60	60	100	100	100
	D	<i>20</i>	20	20	0	40	40	40
	q	<i>0.33</i>	<i>0.33</i>	<i>0.33</i>	0	0.4	0.4	0.4
2	e	<i>1</i>	<i>1</i>	<i>1</i>	∞	1.5	1.5	1.5
	P	<i>40</i>	40	40	40	60	60	60
	D	<i>20</i>	20	20	0	20	20	20
	q	<i>0.5</i>	<i>0.5</i>	<i>0.5</i>	0	0.33	0.33	0.33
3	e	<i>0.5</i>	<i>0.5</i>	<i>0.5</i>	∞	1	1	1
	P	<i>20</i>	20	20	20	40	40	40
	D	<i>20</i>	20	20	0	20	20	20
	q	<i>1</i>	<i>1</i>	<i>1</i>	0	0.5	0.5	0.5
4	e					0.5	0.5	0.5
	P					20	20	20
	D					20	20	20
	Q					1	1	1
Total deaths		100	100	100	0	100	100	100

Note: The new mortality regime starts in year 4. The letter e denotes remaining life expectancy, P population size, D number of deaths and q probability of death. The values in italics pertain to times prior to the mortality shift. The values in bold are discordant with subsequent values prevailing after the shift. Population sizes and death counts for the cohorts born in years 1 and 4 are shaded grey.

6 Discussion

These four simple examples demonstrate how and why mortality change can distort life table statistics. Table 1 shows that if those whose lives are saved gain less additional life expectancy than those whose lives are not saved, then conventional calculations can overestimate true life expectancy. Table 2 shows that if the resuscitated gain more than the remaining life expectancy of those not saved, then conventional calculations can underestimate true life expectancy. Table 3 shows that if the remaining life chances (i.e., future age-specific chances of death) of the resuscitated and the non-resuscitated are the same, then conventional computations result in correct estimates of life expectancy and of age-specific death rates. Finally Table 4 shows that if mortality improvements extend everyone's life by the same amount, a tempo effect, then conventional calculations can result in overestimates of life expectancy. This is explained in detail in my earlier chapter in this volume, but the general point is more clearly illustrated by the simple examples of this chapter.

Note that turbulence in life tables does not depend on a mental model in which the lifespans of individuals are predetermined. The life tables in this chapter could have been described in terms of random hazards of death and I occasionally used this perspective in the explanations above. The crucial question is: what are the age-specific chances of death for those whose lives have been saved as a result of improvements in death rates? If their chances are the same as the chances for those not saved, then conventional calculations are correct. If, however, they have different chances, then conventional life tables are distorted until it is learned, with the passage of time, when the resuscitated die.

Conventional calculations rely on the assumption of homogeneity in life chances for everyone, regardless of whether their lives were saved or not. Bongaarts and Feeney make a very different kind of homogeneity assumption: they postulate that mortality improvements extend everyone's life by the same amount. All populations are heterogeneous and the resuscitated undoubtedly differ from the non-resuscitated and from each other in their remaining life chances. Hence, both conventional calculations and the tempo assumption are wrong. The truth probably lies somewhere in-between.

Some readers may object to my using the phrase "true life expectancy". I define it as the average length of life for a synthetic cohort of newborns who live all their lives under the current mortality regime. An alternative definition would be the average length of life calculated from current age-specific death rates. The second definition, by definition, implies that conventional calculations are correct. This, as I hope the four Tables illustrate, is not an intelligent mode of thinking. The mortality regime in year 4 is the same as thereafter, but life expectancy at birth, in Tables 1, 2 and 4, differs in year 4 from subsequent levels. The distortion is an artifact of mortality change: it does not reflect any underlying differences in mortality regime.

Whether or not tempo effects exist in some age range, at some time periods and in some countries is an open question that merits further research. Whether or not mortality change produces turbulence in lifetables is no longer a question. Research is needed, however, on how much life is extended when a death is averted. The answer surely varies over age, time and place. Only when better knowledge about this is available, will demographers be able to correctly estimate life expectancy and other lifetable statistics in a world of changing mortality.

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