Appendix

Two proofs of a recent formula by Griffith Feeney*

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1 Introduction

In his chapter on increments to life and mortality tempo Feeney gives the following decomposition of the difference between the expectations of life at birth for two cohorts (see also equation (1) on page 154), but without mathematical proof:

$$e_0^c(t_2) - e_0^c(t_1) = -\int_0^\infty \lambda_c^{t_1,t_2}(x)\, dl_c(x,t_1), \tag{1}$$

Since the correctness of this formula was contested during the reviewing process, the editors decided to include the following brief proofs. They are equivalent, but they look different and each may be useful for a different group of readers.

2 Proof by Jutta Gampe

The function $\lambda(x)$ described by Feeney, is given formally as

$$\lambda(x) = l_2^{-1}\left[l_1(x)\right] - x \tag{2}$$

when we drop some obvious subscripts and implicitly assume that everything is invertible etc. Equation (1) can be written either as

$$e_0^2 - e_0^1 = \int_0^\infty \{l_2(x) - l_1(x)\}\ dx$$

or as

$$= \int_0^1 \left\{ l_2^{-1}(p) - l_1^{-1}(p) \right\} \, dp.$$

A change of variables $p \to l_1(x)$ leads to

$$\int_0^\infty \left\{ l_2^{-1} \left[l_1(x) \right] - x \right\} f_1(x) \, dx = \int_0^\infty \lambda(x) f_1(x) \, dx,$$

with $dl_1(x)/\, dx = -f_1(x)$. The latter integral equals

$$- \int_0^\infty \lambda(x) \, dl_1(x)$$

3 Proof by Anatoli Yashin

The condition $l_2(x + \lambda(x)) = l_1(x)$ is equivalent to the condition that the random variables T_2 and $T_1 + \lambda(T_1)$ are identically distributed, hence $ET_2 = ET_1 + E\lambda(T_1)$, which is equation (1).

References

Feeney, G. Increments to life and mortality tempo. *In this volume, also published in Demographic Research*, 14(2):27–46. 2006.