
The quantum and tempo of life-cycle events*

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Summary. This study develops and applies a general framework for the analysis of the period quantum and tempo of life-cycle events, extending methods developed previously by the authors. The existence of tempo distortions is demonstrated in selected period *quantum* measures such as the total fertility rate and in period *tempo* measures such as life expectancy. A tempo distortion is defined as an inflation or deflation of a period quantum or tempo indicator of a life-cycle event, such as birth, marriage, or death, that results from a rise or fall in the mean age at which the event occurs. Period measures derived from life tables are also found to be subject to tempo distortions. Methods to remove these tempo distortions are then developed and applied.

1 Introduction

Questions about human life-cycle events are central to demographic analysis and to social and health policies. How many children do we have? How long do we live? What proportion of men and women ever marry? When do we retire? How much time in old age is spent in good health?

To answer such questions standard demographic methods have been developed to measure key dimensions of the distribution of events over the life cycle. Attention usually focuses on the two primary components of these distributions, the level or *quantum* component and the timing or *tempo* component. Quantum is measured as the average number of events over the course of the life cycle and tempo as the mean age at the event³. The total fertility rate is a quantum measure of fertility, for example, and life expectancy at birth is a tempo measure of mortality.

The quantum and tempo of events can be measured either for *cohorts*, to summarise the actual experience of a group of persons born in the same year,

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³ On the standard hypothetical condition that the cohort experiences no deaths.

or for *periods*, to describe the experience of a hypothetical cohort subject to the conditions observed in a given time period. Cohort measures of quantum and tempo are easily and unambiguously obtained by following a cohort over time until it reaches an age at which the risk of the event equals zero.

Period measures of quantum and tempo, though conceptually more difficult, are far more widely used for two main reasons. First, because cohort indicators measure ongoing changes in demographic processes after a lag, they cannot adequately describe year-to-year changes. Second, period measures require less historical data than cohort measures and may therefore be calculated for many more countries and more times.

This study continues our work on tempo effects in demographic analysis. Bongaarts and Feeney (1988a) defines period tempo distortions for total fertility rates and provides a method for correcting these distortions. Bongaarts and Feeney (2002, in this volume p. 11) gives analogous results for life expectancy. This chapter develops and applies a general framework for the analysis of the period quantum and tempo of life-cycle events of all kinds, with empirical examples for fertility, marriage and mortality. We begin with a brief presentation of the two main types of age-specific rates from which period quantum and tempo measures are calculated. The remainder of the study is divided into two parts corresponding to the two types of rates. Both of these parts demonstrate the existence of tempo distortions in selected period quantum and tempo measures. A tempo distortion is defined as an inflation or deflation of a period quantum or tempo indicator of a life-cycle event, such as birth, marriage, or death, that results from a rise or fall in the mean age at which the event occurs. We then develop and apply methods to remove these tempo effects.

2 Background: age-specific event rates

Two types of age-specific rates are used in demographic analysis: rates *of the 1st kind*, or *hazard rates*; and rates *of the 2nd kind*, or *incidence rates* (Henry 1972; Sobotka 2003, 2004a; Kohler and Ortega 2002a).

Rates of the 1st kind (hazard rates) are illustrated by standard age-specific death rates. They are quotients in which the numerator counts events occurring to persons at age a and time t and the denominator counts persons *exposed to risk* of the event in question at age a and time t . These rates are also called risks, intensities, conditional rates and occurrence/exposure rates. For the life-cycle events considered in this chapter—first birth, first marriage, and death—persons exposed to risk are those who have not already experienced the event.

Rates of the 2nd kind (incidence rates) are illustrated by standard age-specific birth and marriage rates. They are quotients in which the numerator counts events occurring to persons at age a and time t and the denominator counts *all persons* at age a and time t , including those who have already

experienced the event. Incidence rates are also called densities, unconditional rates, reduced rates, and frequencies.

The relation between rates of the 1st kind and rates of the 2nd kind is straightforward for first births, first marriages, and death. The denominators of hazard rates exclude persons who have already experienced the event, whereas the denominators of rates of incidence include these persons. This relationship may be expressed using life table notation as $\mu(a, t) = d(a, t)/p(a, t)$, where $\mu(a, t)$ denotes a hazard rate, $d(a, t)$ a corresponding incidence rate, and $p(a, t)$ denotes the proportion of persons born at time $t - a$ who have not experienced the event by age a .

The relation between the two kinds of rates for events that can occur more than once in a lifetime (recurrent events) is more complicated. The established way of dealing with recurrent events is to number events in order of occurrence to each individual. For example, births are divided into first births, second births, third births, and so on. In this way any recurrent event may be resolved into a series of non-recurrent events, which can be analysed separately.

Table 1 displays and compares the two kinds of rates for first birth, first marriage, and death. The first row shows clearly the distinction between rates of the 1st and 2nd kind. The numerators of the two rates are the same (first births), but the denominators of rates of the 1st kind exclude women who have already had a first birth, whereas the denominators of rates of the 2nd kind include these women. Summing these rates of the 2nd kind for all birth orders gives the standard age-specific birth rates from which the total fertility rate is calculated.

The second row of Table 1 shows the two kinds of rates for first marriages. As in the case of first births, the numerators of the two rates are the same. The denominators of rates of the 1st kind exclude women who have already married, whereas the denominators of rates of the 2nd kind include these women.

The last row of the table shows the two kinds of rates for death. The rates of the 1st kind are standard age-specific death rates or, given the continuous formulation, the force of mortality. As in the case of the rates for first birth and first marriage, the numerators of the two kinds of death rates are the same, but the denominators of the rates of the 1st kind exclude persons who have already experienced the event—i.e., persons who have died—whereas the denominators of rates of the 2nd kind include these persons. Thus the denominators of the death rates of the 2nd kind include persons in the cohort who have already died as well as those who are living.

Death rates of the 2nd kind are obviously unconventional, for although the quotient shown is a standard demographic statistic—the value of $d(x)$ in the cohort life table for persons born at time $t - a$ —this statistic has not generally been regarded as comparable to the other frequencies shown in the table. Indeed, it is only regarded in this way in a very few studies, e.g., by Sardon (1993, 1994) and Bongaarts and Feeney (2002, in this volume p. 11).

Table 1. Rates of the 1st and 2nd kind for first birth, first marriage, and death.

Event	Rates of the 1st kind (occurrence-exposure rates)	Rates of the 2nd kind (frequencies)
First birth	1st births at age a and time t	1st births at age a and time t
	Childless women age a at time t	All women age a at time t
First marriage	1st marriages at age a and time t	1st marriages at age a and time t
	Never-married women at age a and time t	All women age a at time t
Death	Deaths at age a and time t	Deaths at age a and time t
	Persons living at age a and time t	All persons born at time t a

Death rates of the 2nd kind are strictly analogous to first birth rates and first marriage rates of the 2nd kind. For all three events, the denominator includes persons who have not yet experienced the event as well as persons who have already experienced the event. The characterisation of the denominator for death rates of the 2nd kind appears exceptional only because “persons” usually connotes “living persons”, though of course it may refer to deceased persons as well.

An important general property of rates of the 2nd kind for non-repeatable life cycle events is that the sum (integral) of these rates over all ages for a birth cohort gives the proportion of the cohort that experiences the event. Thus summing age-specific first birth rates gives the proportion of women ever having a (first) child, and summing age-specific first marriage rates gives the proportion ever marrying. The sum of the death rates of the 2nd kind over all ages for a birth cohort will equal one because everyone dies.

The interpretation of the sum of rates of the 2nd kind over all ages as the proportion experiencing the event is straightforward and unambiguous for *cohorts*. Calculating the sum of first birth, first marriage or death rates for *periods* is equally straightforward, but the interpretation of these sums as the proportions ever experiencing the event in synthetic cohorts is problematic due to tempo effects in period measures, as will be demonstrated next.

3 Period quantum and tempo measures of the 2nd kind

3.1 Standard equations for quantum and tempo measures

Table 2 presents general equations for calculating period quantum and tempo from rates of the 2nd kind and specific results for first birth, first marriage, and death. The total event rate $TER(t)$ and the mean age at event $MAE(t)$ are defined by the formulas in the first row of the table, with $d(a, t)$ denoting the age-specific rate of the 2nd kind for any of the events shown. The total event rate equals the average number of events over the life cycle for a hypothetical cohort subjected to the rates at time t (in the absence of competing events).

For events that occur only once, the total event rate equals the proportion of persons in the hypothetical cohort who ever experience the event.

The quantum and tempo measures of first birth and first marriage in Table 2 are standard tools in demographic analysis and estimates are available for many countries. The total mortality rate and the mean age at death (birth cohort normalised), though defined in precise analogy with the fertility and mortality measures, were introduced for the first time by Sardon (1993, 1994) and further analysed in Bongaarts and Feeney (2002, in this volume p. 11; note that $MAD(t)$ is not the mean age of deaths occurring at time t because the effects of variations in cohort size are removed). The table therefore illustrates that measures that are standard for some demographic processes may be unknown in the study of other processes.

Table 2. Period measures of quantum and tempo based on rates of the 2nd kind for first birth, first marriage, and death.

	Period quantum
General formula	Total event rate, $TER(t)$ $TER(t) = \int_0^\infty d(a, t) da$
First birth	Total fertility rate, order 1, $TFR_1(t)$
First marriage	Total 1st marriage rate, $TNR_1(t)$
Death	Total mortality rate, $TMR(t)$
	Period tempo
General formula	Mean age at event, $MAE(t)$ $MAE(t) = \frac{1}{TER(t)} \int_0^\infty ad(a, t) da$
First birth	Mean age at 1st birth, $MAB_1(t)$
First marriage	Mean age at 1st marriage, $MAM_1(t)$
Death	Mean age at death (birth cohort normalised), $MAD(t)$

Note: " TNR " (N for "nuptiality") is written for 1st marriage so that " TMR " may be used for Total mortality rate.

Figures 1 to 3 present empirical results for the quantum and tempo measures summarised in Table 2 for selected populations. Figure 1 shows total fertility rates for birth order one, $TFR_1(t)$, and the mean age at first birth, $MAB_1(t)$, for the United States from 1950 to 2000. Values of $TFR_1(t)$ exceeded one for most of the 1950s, an obvious anomaly since no woman can have more than one first birth. This period of elevated fertility coincided with the decline in the age at first birth during the baby boom years of the 1950s.

Figure 2 shows total first marriage rates, $TNR_1(t)$, and the mean age at first marriage, $MAM_1(t)$, for France from 1960 through 2001. The above-one rates for France in the early 1960s are anomalous because a woman can experience at most one first marriage. The apparent explanation, by analogy with that for first births, is the declining mean age at first marriage. First marriage rates for France decline over the period shown, with values around 0.5 toward the end of the period. Similar trends are observed in many other

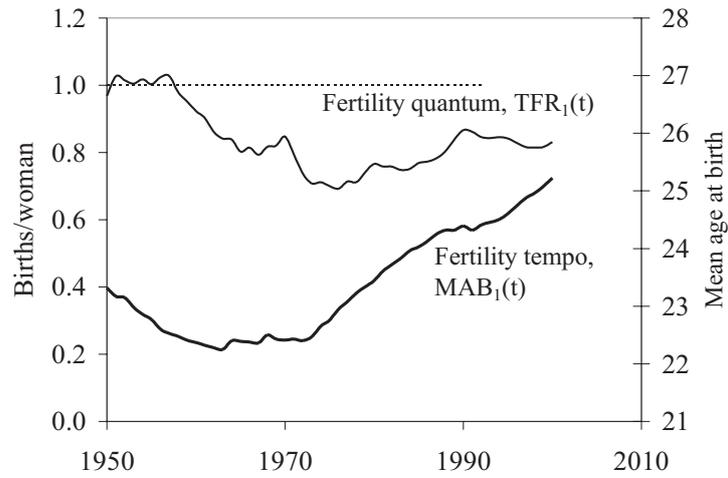
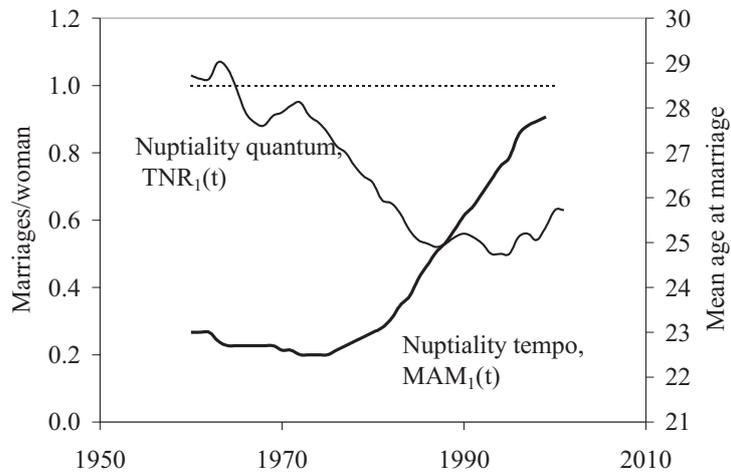


Fig. 1. Total fertility rate, order one, and mean age at first birth in the USA.

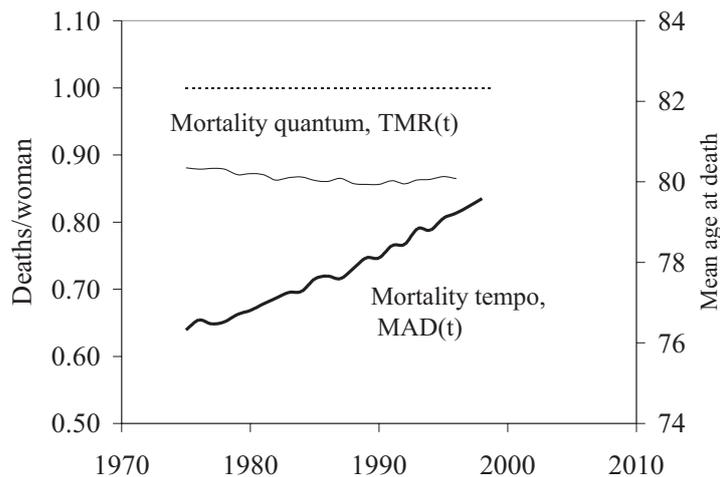
European countries, but proportions ever-married for cohorts born in the late 1960s are much higher than 0.5 (Council of Europe 2002). This suggests that the low first marriage rates are distorted.



Source: Council of Europe 2002

Fig. 2. Total first marriage rate and mean age at first marriage, females in France.

Figure 3 shows total mortality rates, $TMR(t)$, and the mean age at death, $MAD(t)$, for England and Wales from 1975 to 1998. (For reasons given below all mortality measures in this study include adult mortality above age 30 only.) The total mortality rate is well below one (0.85-0.90). Since every person dies once, any total mortality rate other than one is anomalous. Mortality tempo (MAD) rose sharply throughout the period, and the analogy for first birth and first marriage therefore suggests again that this is the reason for the TMR values different from one.



Source: Estimated from data in Human Mortality Database (2005)

Fig. 3. Total mortality rate and mean ages at death, females in England and Wales (adult mortality only).

3.2 Tempo effects

We will now demonstrate that the various anomalies evident in Figures 1, 2, and 3 are largely attributable to tempo effects. A tempo effect is defined as an inflation or deflation of the number of events observed in a period when the period (cohort size adjusted) mean age changes. Tempo effects in event numbers lead to tempo effects in event rates (of the first and second kind) and these in turn lead to tempo effects in most period tempo and quantum measures. Tempo effects in these aggregate measures (but not in rates) will also be referred to as distortions, following terminology introduced by Ryder (1956) in his analysis of the fertility tempo effect. This section presents

the theoretical basis for this effect and offers additional empirical evidence supporting the theory.

Theoretical basis for tempo effects

Norman B. Ryder (1956, 1959, 1964, 1980, 1983) made a series of fundamental contributions to the study of quantum and tempo measures. His paradigmatic contribution was a simple model that showed that the period total fertility rate (*TFR*) does not, in general, equal the cohort completed fertility rate (*CFR*) even if fertility has been constant for a long period of time. His “translation” formula

$$TFR = CFR(1 - r_c) \quad (1)$$

shows that the *TFR* in a constant fertility population tends to be lower than the *CFR* when the cohort mean age at childbearing is rising (i.e., the rate of change in this mean, r_c , is positive and hence $(1 - r_c) < 1$) and higher than the *CFR* when the mean age at childbearing is falling (r_c is negative, $(1 - r_c) > 1$). This equation assumes linearity in time trends of the age-specific fertility rates. Ryder refers to $(1 - r_c)$ as an “index of fertility distortion” and he considered the *TFR* to be a distorted measure when the fertility tempo changes.

Ryder’s analyses of period fertility trends in the United States (1980, 1983) showed how changes in the timing of childbearing among cohorts of women influenced annual age-specific birth rates and total fertility rates. When women shift upward the ages at which they bear children, annual numbers of births tend to be deflated because the same number of births will be spread out over a longer time period (e.g., during the 1970s and 1980s). Similarly, when age at childbearing shifts to younger ages, total fertility rates tend to be inflated because the same number of births are compressed into a shorter time period (e.g., during the late 1940s and 1950s).

Zeng and Land (2002) extend Ryder’s analysis by deriving the following translation formula,

$$TFR = CFR(1 - r_p), \quad (2)$$

where r_p denotes the rate of change in the period mean age at childbearing and *TFR*, *CFR*, r_p , and the shape of the schedule of age-specific fertility rates are assumed constant. They consider this alternative version of the translation equation preferable to (1) because their constant shape assumption is more realistic than Ryder’s linearity assumption.⁴ The conditions under which (2) holds (i.e., constant quantum, fixed rate of increase in the period mean, and an invariant shape) will collectively be referred to as the “translation assumptions”.

⁴ Zeng and Land (2002) prove this with their assumptions $TFR = CFR/(1 + r_c)$ and $r_c = r_p/(1 - r_p)$, where r_p is the rate of change of the period mean age of childbearing. Cf. formula (2).

These translation equations were developed for the analysis of fertility trends, but analogous equations apply to other life-cycle processes provided that the same translation assumptions apply. For mortality, for example, we have

$$TMR = CMR(1 - r_p), \quad (3)$$

where TMR is the total mortality rate, CMR denotes the cohort completed mortality rate, and r_p is the rate of change in the period mean age at death, MAD (see Table 2). Because everyone dies once, the CMR equals (1) and (3) simplifies to

$$TMR = 1 - r_p \quad (4)$$

This result shows the operation of the tempo distortion in its most basic form: the TMR simply equals the distortion index. The undistorted value of $TMR = 1$ is obtained only if the mean age at death is constant (i.e., $r_p = 0$). Any change in the mean age at death, whether up or down, results in a tempo effect in the total mortality rate and in the mortality rates of the 2nd kind from which it is calculated. The effect is evident in Figure 3, which shows that estimates of TMR for England and Wales is about 0.86. This is more or less consistent with the rate of change in the mean age at death shown in Figure 3, about 0.14 years per year.

The period-cohort translation formulas of Ryder (1) and Zeng and Land (2) may be applied when fertility is changing slowly by comparing the TFR for any given year with the CFR for the cohort that reaches its mean age at childbearing in this year (Ryder 1956; Sobotka 2003). If the 1960 birth cohort has a mean age at first birth of 25 years, for example, the CFR for this cohort is compared with the TFR for 1985. To attenuate year-to-year fluctuations, $TFRs$ may be averaged over a series of years.

To illustrate, Table 3 presents evidence for tempo distortion in the TFR of France during the last quarter of the 20th century. The TFR was relatively stable during this period, with an average value of 1.80 children per woman. The completed fertility rate (CFR) for the cohorts that were at prime childbearing ages during these periods was also nearly stable, but with an average value of 2.08 children per woman, 0.28 children per woman higher than the average total fertility rate.

This disparity between period and cohort fertility is explained largely by a tempo distortion on the TFR resulting from the change in the period mean age at childbearing. This mean rose at an average annual rate of 0.125 years per year between 1975-80 and 1995-99 ($r_p = 0.125$). Since the constant fertility assumption is approximately valid, the TFR implied by (2) is $CFR(1 - r_p) = 2.08(1 - 0.125) = 1.82$ births per woman. This is very close to the observed average total fertility rate for the period, 1.80 children per woman. In this example, the translation formula (2) quite accurately estimates the tempo distortion due to rising mean age at childbearing.

In this illustration, the translation formula has been applied to births of all orders. In general, however, it is recommended that the translation formula

Table 3. Analysis of tempo distortion of the period total fertility rate (*TFR*) in France, 1975-1999.

Period	<i>TFR</i> (births per woman)	<i>CFR</i> (births per woman) ^a	Mean age at child-bearing (years)
1975-79	1.86	2.11 (1950) ^a	26.6
1980-84	1.88	2.13 (1955)	27.1
1985-89	1.81	2.10 (1960)	27.9
1990-94	1.72	1.99 (1965)	28.5
1995-99	1.74		29.1
Average	1.80	2.08	

Source: Council of Europe 2002.

Note: a Year of birth of cohort in parentheses.

be applied separately for births of each order, as illustrated in Bongaarts and Feeney (1998a).

Empirical evidence supporting the theory: first births, first marriages, and deaths

This section systematically applies and tests the Zeng-Land translation formula (2) using empirical data for first birth, first marriage, and death.

-*First births.* Figure 4 compares completed first birth cohort fertility for women born in 1960 (CFR_1) and period first birth total fertility for 1980-89 (TFR_1) for 15 European countries, the USA, and Japan. In most countries, the cohort level exceeds the period level. To show that this difference is due largely to tempo distortions, the translation equation (2) is rearranged as follows:

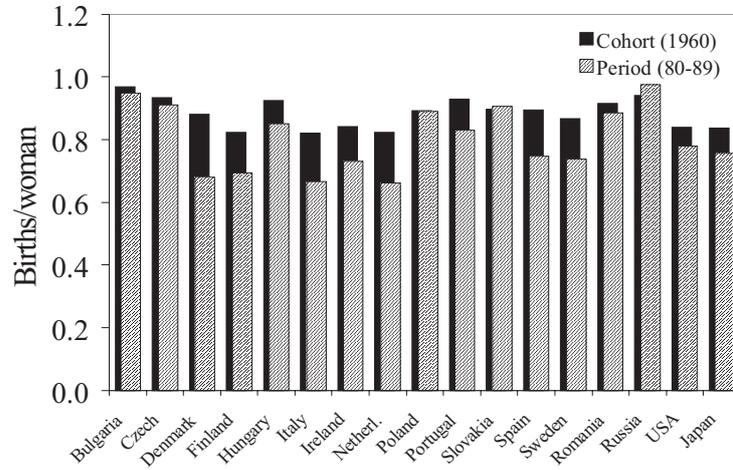
$$\frac{TFR}{CFR} = 1 - r_p. \quad (5)$$

This shows that, in a constant fertility population, there is a simple linear relationship between TFR/CFR and $(1 - r_p)$, so that if values of these two quantities for different countries are scatter plotted, the points will lie on a straight line with slope one that passes through the origin. We refer to $(1 - r_p)$ as the *period distortion index*.

To test the validity of this translation equation for first births, Figure 5 plots TFR_1/CFR_1 ratios (vertical axis) against the corresponding values (horizontal axis)⁵ for the 17 countries represented in Figure 4. There is a close correspondence between the data points for the 17 countries and the linear relation predicted by the translation equation. This is consistent with tempo distortions of the TFR_1 being the main explanation for the difference between the TFR_1 and the CFR_1 . We do not expect the observations for the different

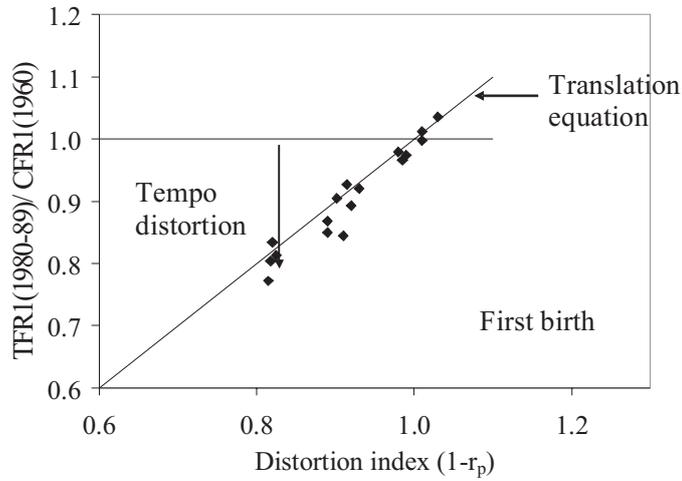
⁵ The TFR value is the average TFR_1 for 1980-89. The CFR value is CFR_1 for the 1960 birth cohort. Values of r_p are estimated as one tenth of the difference between the period mean ages at first birth in 1980 and 1990.

countries to fall exactly on the diagonal because the translation assumptions hold only approximately.



Source: Bongaarts 2002

Fig. 4. Completed cohort fertility (1960) and period total fertility(1980-89), first births.



Source: Bongaarts 2002

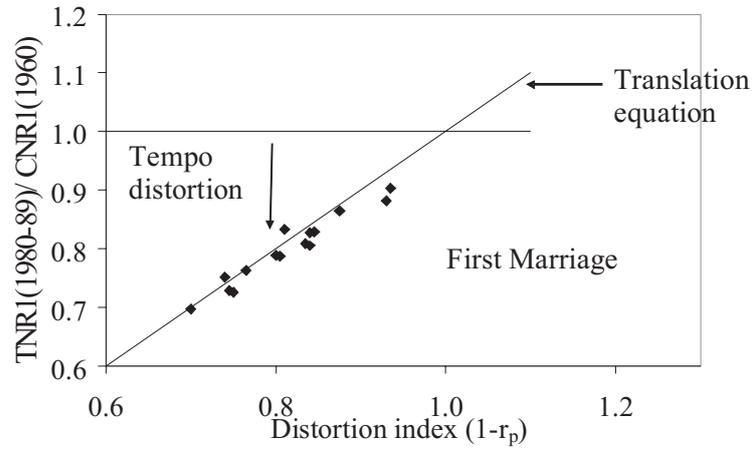
Fig. 5. Ratio of period to cohort fertility rate by tempo distortion index, first births, 17 countries.

-*First marriages.* The same translation formula analysis may be applied to quantum and tempo measures of first marriage. The total first marriage rate is influenced by tempo effects in the same way that the total fertility rate is. When the mean age at marriage is rising (falling), the same number of marriages occur over a longer (shorter) period and annual numbers of marriages are lower (higher) than they would have been in the absence of the change in mean age. Most of the concepts and derivations developed for the analysis of fertility tempo apply to the analysis of “nuptiality tempo” as well. Recent studies by Goldstein (2003) and by Winkler-Dworak and Engelhardt (2004) provide examples of this application.

Figure 6 tests the translation equation for nuptiality. The ratio of the period to cohort quantum for first marriage (TNR_1/CNR_1) is plotted against the distortion index $(1 - r_p)$, with r_p representing the rate of change in period mean age at first marriage. The diagonal line represents the relationship predicted by the translation equation. Most countries again fall close to the predicted values, confirming the existence of tempo effects for first marriage rates.

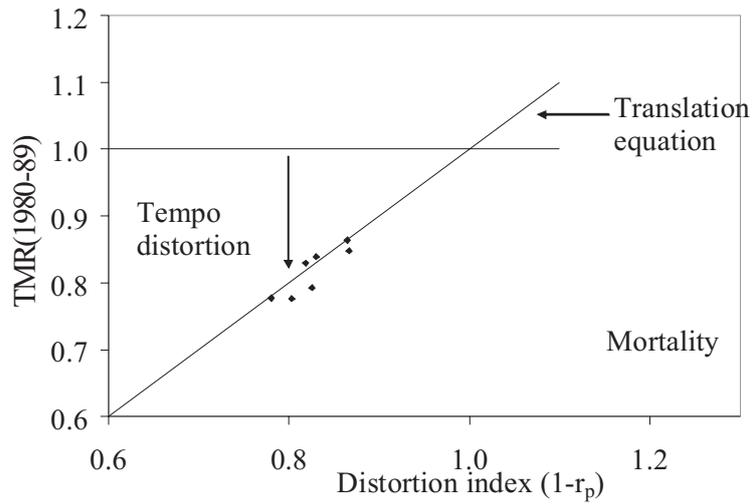
-*Deaths.* We now extend the same translation formula analysis to the quantum and tempo measures of mortality based on rates of the 2nd kind. Since the cohort completed mortality rate necessarily equals one, the period-cohort ratio equals the *TMR*. The relationship predicted by the translation equation (4) is given by the diagonal line in Figure 7. (Following Bongaarts and Feeney (2002, in this volume p. 11) the analysis of the quantum and tempo of mortality is limited to adult mortality above age 30 to ensure consistency with the constant shape assumption.) Figure 7 includes the resulting data points for seven countries (England and Wales, Italy, France, Norway, Switzerland, Sweden, and the US) for which the required historical data from 1900 to the present are available. As in the fertility and nuptiality analyses, the data points fall close to the line predicted by the translation equation, supporting both the validity of the translation equation and the existence of tempo distortions for adult mortality.

These analyses show that the distortions established in the case of fertility apply to nuptiality and (adult) mortality as well when the period quantum measures for each event are calculated from rates of the 2nd kind. They also show that the magnitude of tempo distortions may be substantial. Figures 5-7 show that average distortions of 10 per cent are common during the 1980s and that distortions exceeding 20 per cent occur for some countries for fertility, mortality, and in particular for nuptiality. The distortions are even larger in individual years. Bongaarts and Feeney (1998a), for example, estimate distortions in the *TFR* in the United States ranging from +28% in 1948 to -11 % in 1975.



Source: Council of Europe 2002

Fig. 6. Ratio of period to cohort marriage rate by tempo distortion index, first marriages, 17 countries.



Source: Estimated from data in Human Mortality Database 2005

Fig. 7. Period total mortality rate by tempo distortion index, in 7 countries.

3.3 Correcting tempo distortions in quantum measures of the 2nd kind

Ryder's work established the existence of tempo distortions in the total fertility rate, but he did not propose specific, quantitative adjustments to counteract tempo distortions. This may be explained in part by his strong emphasis on the conceptual priority of cohort fertility. The emphasis on cohorts probably influenced his focus on "translating" period measures to cohort measures as well, which diverted attention from the problem of adjusting period measures for tempo distortions.

Empirical research over the past three decades has demonstrated, however, that period influences on fertility are much more important than cohort influences. Brass (1974) concludes that cohort completed fertility reveals no significant feature that distinguishes it from time averages of period indexes. Pullum (1980) concludes that "temporal variations that cut across cohorts, such as economic cycles, appear to be more important than changes in those variables that distinguish cohorts, such as shared socialising experiences" (see also Page 1977). Foster's (1990) analysis of data for eight countries in Europe and North America arrives at a similar conclusion. In an authoritative review, N Bhrolchin (1992) concludes that "of the two dimensions of calendar time-period and cohort-period is unambiguously the prime source of variation in fertility rates." Bongaarts and Feeney (in this volume p. 11) demonstrate that the same dominance of period effects exists for adult mortality rates in contemporary populations with high life expectancy. These findings provide the basis for the tempo adjustment procedure discussed next.

Correcting tempo distortions in period quantum measures of fertility

Bongaarts and Feeney (1998a) reformulated the issue of tempo distortions by posing the following counterfactual question: What would the total fertility rate have been in a particular year, other things being equal, if the mean age at childbearing had been constant during that year? The purpose is to remove the distortion resulting from a changing mean age of the event to obtain a better measure of current fertility conditions. Subject to a simplifying assumption on the pattern of fertility change, they show that the answer to this question is given by

$$TFR^*(t) = \frac{TFR(t)}{1 - r_p(t)} \quad (6)$$

where r_p denotes the rate of change in the period mean age at childbearing in year t . $TFR^*(t)$ is referred to as the *tempo-adjusted TFR*, and the tempo distortion in the observed TFR equals $TFR^*(t) - TFR(t)$.

Unlike the translation formulas (1) and (2), formula (6) involves only period measures. Another advantage of (6) is that it separates the issue of tempo

distortion from the question of the relationship between period and cohort measures. The tempo-adjusted is not intended to estimate and need not equal the *CFR* for any cohort. (However, as shown in Appendix A, the *CFR* is approximately equal to the weighted average of values observed during the years in which the cohort reproduces.)

Formula (6) depends on the *constant shape assumption*, which may be stated in this way: the age schedule of fertility rates (of the 2nd kind) observed at any time can be transformed into the schedule observed at any other time by inflating or deflating and/or by shifting the schedule to higher or lower ages. This is equivalent to assuming that fertility is determined strictly by period effects. By comparison, formula (2) requires not only the constant shape assumption, but also (page 9) constant quantum and a constant rate of increase of the mean age. The tempo adjustment equation (6) can therefore be applied much more widely. Recent studies by Kohler and Philipov (2001) and Zeng and Land (2001) confirm the mathematical derivation of (6).

Although equation (6) can be applied to births of all orders combined, superior results are obtained by applying the formula separately to each birth order component of the *TFR*, because the constant shape assumption is more valid for the fertility schedule at each order than for all orders combined (Bongaarts and Feeney 1998a). This disaggregation is particularly important in countries in which the overall *TFR* is changing rapidly, for this is likely to result in substantial changes in the weighting of the different birth order components.

Bongaarts and Feeney (1998a) and the follow-up work of Bongaarts (1999a, 1999b, 2002) have stimulated a number of criticisms, extensions, and elaborations. Van Imhoff and Keilman (2000) and Van Imhoff (2001) point out that the constant shape assumption does not hold exactly for the Netherlands and Norway during the second half of the 20th century. This issue is addressed by Zeng and Land (2001), who carried out a sensitivity analysis and concluded that “the Bongaarts-Feeney formula is not sensitive to temporal changes in the shape of the fertility schedules”. Kohler and Philipov (2001), on the other hand, find that errors resulting from deviations from the assumption in Sweden were not insignificant and addressed this by proposing a procedure for calculating tempo-adjusted total fertility rates when the variance of the fertility schedule changes over time (see also Kohler and Ortega 2002a and 2002b). A number of past studies have applied the adjustment to fertility (Lesthaeghe and Willems 1999; Smallwood 2002; Sobotka 2003, 2004a, 2004b). Implications of fertility tempo effects for population growth are examined by Goldstein et al. (2003).

Extension of tempo adjustments to nuptiality and mortality

The Bongaarts-Feeney method can be extended to obtain estimates of tempo-adjusted period quantum measures for life-cycle events other than fertility. Table 4 shows formulas for adjusted quantum and tempo for the total first

birth rate, the total first marriage rate, and the total mortality rate. Tempo adjustments are effected by dividing observed event rates of the 2nd kind by the period tempo distortion index, as in formula (6) above. As in Table 2, the formulas in the first row of the table define the tempo-adjusted total event rate $TER^*(t)$ and the tempo-adjusted mean age at event $MAE^*(t)$, with $d(a, t)$ denoting the age-specific rate of the 2nd kind for any of the events shown.

The tempo-adjusted total mortality rate TMR^*

$$TMR^*(t) = \frac{TMR(t)}{1 - r_p} \quad (7)$$

is of particular interest. Because $TMR^*(t)$ must equal one, it follows that

$$TMR(t) = 1 - r_p(t) \quad (8)$$

This is a more general version of (4) because it allows $TMR(t)$ and $r_p(t)$ to vary over time. As noted, the results in Figure 7 confirm this relationship for mortality over age 30.

The right hand column of Table 4 shows that tempo measures based on rates of the 2nd kind are not affected by tempo distortion (assuming the constant shape assumption holds). This is because the distortion index occurs in both the numerator and the denominator of the formula, and so cancels out. Empirical confirmation of this conclusion will be provided in a later section.

Table 4. Adjustments for tempo distortions in period quantum and tempo measures based on rates of the 2nd kind.

Event	Adjusted quantum (total event rate)	Adjusted tempo (mean age at event)
General	$TER^*(t) = \int_0^\infty \frac{d(a,t)}{1-r_p(t)} da$ $= \frac{TER(t)}{1-r_p(t)}$ $r_p(t) = \frac{dMAE(t)}{dt}$	$MAE^*(t) = \frac{1}{TER^*(t)} \cdot \int_0^\infty \frac{ad(a,t)}{1-r_p(t)} da$ $= MAE(t)$
First birth	Tempo-adjusted total fertility rate, order 1, $TFR_1^*(t)$	No adjustment needed: $MAB_1^*(t) = MAB_1(t)$
First marriage	Tempo-adjusted total first marriage rate $TNR_1^*(t)$	No adjustment needed: $MAM_1^*(t) = MAM_1(t)$
Death	Tempo-adjusted total mortality rate $TMR_1^*(t)$	No adjustment needed: $MAD^*(t) = MAD(t)$

Empirical application to first births, first marriages, and deaths

Empirical estimates of tempo-adjusted quantum measures contain seemingly random year-to-year fluctuations. These are caused by sensitivity to small errors in $r_p(t)$ and by deviations from the constant shape assumption. To

minimise these fluctuations, we plot five-year moving averages of $TER^*(t)$ in place of annual values in Figures 8-10.

Figure 8 presents observed and tempo-adjusted total first birth fertility rates for the United States⁶. The adjusted rates are lower than the observed rates during the 1950s and early 1960s, when the mean age at first birth was declining, and higher than the observed rates between 1975 and 1990, when the mean age at first birth was rising.

Figure 9 presents observed and adjusted total first marriage rates for France from 1960 to 1996. The adjusted rates are more plausible than the observed rates, because the adjusted rates are uniformly below one and because the tempo-adjusted total first marriage rate *circa* 1990 is 0.7, which is approximately equal to the proportion ever marrying among cohorts born in the late 1960s.

Figure 10 gives the observed and adjusted total mortality rate (adult mortality only) for England and Wales. The adjusted rate fluctuates around to the level of one, as it should. If the constant shape assumption holds perfectly, the tempo-adjusted *TMRs* would all equal one exactly, assuming no measurement error.

Figures 8-10 also include the corresponding quantum estimates for successive cohorts. Let $M(c)$ be the mean age at the event for the cohort born in year c . In Figure 8 the cohort quantum (i.e., the proportion ever having a first birth) for the cohort born in year $c = t - M(c)$ is plotted at time t . In Figure 9 this comparison of cohort and period quantum is made for the first marriage quantum and in Figure 10 for the quantum of mortality. In each of these comparisons, the (lagged) cohort quantum is close to the adjusted rate of the 2nd kind, but the fit is not perfect. Perfect agreement between the cohort and adjusted period quantum is expected only when all the translation assumptions hold: the period and cohort quantum are constant, the period mean age is rising linearly, and the shape of the age pattern is constant. Since these assumptions do not hold exactly, the cohort quantum is not exactly equal to the adjusted total event rates, but the correspondence is good and clearly better than for the unadjusted rates. The observed differences between the cohort quantum and adjusted period quantum are due to three factors: deviation from the constant quantum assumption, deviation from the linear change assumption, and deviation from the constant shape assumption. The first two of these deviations do not cause errors in the adjusted quantum, which only requires the constant shape assumption. As a result even when

⁶ Application of the tempo-adjustment formulas requires annual estimates of the total event rate and the rate of change in the period mean age of the event. The rate of change during year t is estimated as $0.5[MAE(t+1) - MAE(t-1)]$. Application of formulas in Table 4 then gives the time series of tempo-adjusted total event rates. Since the adjustment is sensitive to small errors in $r_p(t)$, the annual adjusted estimates tend to contain seemingly random fluctuations. To minimise these fluctuations, we use five-year moving averages of $TER^*(t)$ in place of annual values.

the adjusted quantum is accurately estimated, it can differ from the lagged cohort quantum.

In the applications summarised in Figures 8-10 the adjustment procedure appears to work well. The obvious anomalies in unadjusted quantum measures noted earlier are all removed by the tempo adjustment, and the adjusted quantum is close to the lagged cohort quantum, even though the conditions for this comparison are not fully met.

Quantum and tempo measures of the 2nd kind are much more widely used in the analysis of fertility and nuptiality than measures of the 1st kind, because age-specific rates of the 2nd kind (adjusted and unadjusted) are easier to calculate and more widely available. A disadvantage of rates of the 2nd kind is that they can be affected by compositional effects (Kohler and Ortega 2004). It is therefore generally considered preferable to derive quantum and tempo measures from rates of the 1st kind if such rates are available. As we will show below, however, these rates are affected by tempo effects, and therefore need adjustment.

4 Period quantum and tempo measures of the 1st kind

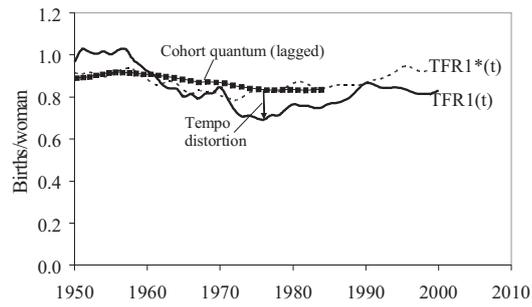
The preceding sections of this study examined quantum and tempo measures of the 2nd kind as well as tempo effects in these measures. The present section will cover these same topics for rates of the 1st kind. These rates are used extensively in life table analyses of the quantum and tempo of life-cycle events. This discussion will be briefer because the main concepts have already been introduced and because data on measures of the 1st kind are not widely available except for mortality.

4.1 Standard equations for quantum and tempo measures

Table 5 presents equations for estimating quantum and tempo measures derived from rates of the 1st kind⁷. Applying the general formulas in the first row to first birth, first marriage, and death produces quantum estimates TFR_{1L} , $TNR_{1L}(t)$, and $TMR_L(t)$ and tempo estimates $MAB_{1L}(t)$, $MAM_{1L}(t)$, and $MAD_L(t)$. The subscript L signifies that these measures are based on the life table calculation using rates of the 1st kind. This distinguishes them from the corresponding measures based on rates of the 2nd kind (see Table 2).

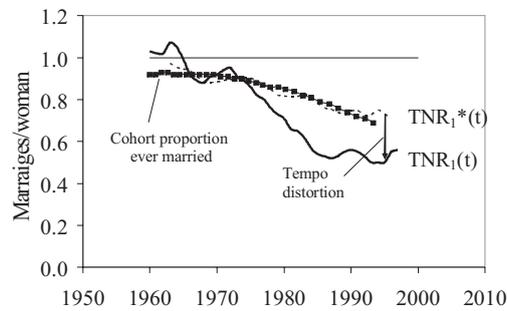
Period quantum based on rates of the 1st kind is defined as the proportion of persons ever experiencing the event in a hypothetical cohort subjected to these rates, as given by the standard life table calculation. The quantum of mortality $TMR_L(t)$ necessarily equals one because everyone eventually

⁷ The first formula in the first line of Table 5 shows that $TMR_L(t)$ equals one minus an integral. This integral equals the proportion of individuals that never experiences the event.



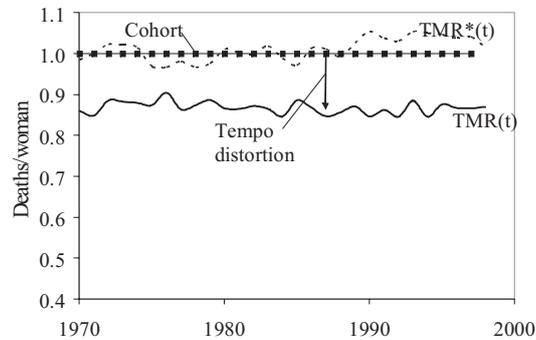
Source: Bongaarts and Feeney, 1998

Fig. 8. Observed and tempo adjusted total fertility rate, birth order one, United States.



Source: Council of Europe 2002

Fig. 9. Observed and tempo adjusted total first marriage rate, females, France.



Source: Estimated from data in Human Mortality Database

Fig. 10. Observed and tempo adjusted total mortality rate, females, England and Wales.

dies. The quantum of first birth $TFR_{1L}(t)$ and the quantum of first marriage $TNR_{1L}(t)$ are less than one because the rates from which they are calculated fall rapidly to zero at older ages.

The most widely used period tempo measure of the 1st kind is life table mean age at death $MAD_L(t)$, which is usually referred to as life expectancy. With $TMR_L(t) = 1$ the general tempo equation on the right in Table 5 simplifies to

$$MAD_L(t) = e_0(t) = \int_0^\infty e^{-\int_0^a \mu(x,t) dx} da, \tag{9}$$

which is the conventional expression for the life table estimate of life expectancy at birth conventionally denoted $e_0(t)$.

Cohort measures based on rates of the 1st kind are identical to the corresponding measures based on rates of the 2nd kind, but period measures based on rates of the 1st kind do not in general equal the corresponding measures based on rates of the 2nd kind. This will be illustrated in the following section.

Table 5. Adjustments for tempo distortions in period quantum and tempo measures based on rates of the 2nd kind.

Event	Quantum (total event rate = proportion ever experiencing event)	Tempo (mean age at event)
General	$TER_L(t) = 1 - e^{-\int_0^\infty \mu(a,t) da}$	$MAE_L(t) = \frac{1}{TER_L(t)} \int_0^\infty e^{\int_0^a \mu(x,t) dx} da + \int_0^\infty TER_L(t) - 1 da$
First birth	$TFR_{1L}(t)$	$MAB_{1L}(t)$
First marriage	$TNR_{1L}(t)$	$MAM_{1L}(t)$
Death	$TMR_L(t)$	$MAD_L(t)$

4.2 Tempo effects

Tempo effects result from a depression or inflation in the numbers of events that occur in the numerators of rates. These effects therefore affect age-specific rates of the 1st kind as well as rates of the 2nd kind. Moreover, the effect is proportionally the same for the numerators of age-specific rates of the 1st and 2nd kind and it is determined by the distortion index, which varies with the rate of change in the mean age at the event. This point was first made by Bongaarts and Feeney (1998a) and subsequently by Kohler and Ortega (2002a, 2002b) in their analysis of tempo effects in fertility rates of the 1st kind.

Tempo distortions of measures of the 1st kind are generally less noticeable than distortions of measures of the 2nd kind. Tempo distortions in quantum

measures based on rates of the 2nd kind are obvious, for example, whenever these measures exceed one. This cannot happen for quantum measures based on rates of the 1st kind because the life table calculations used necessarily lead to values less than or equal to one. The absence of obvious anomalies in these measures does not mean that they are free of tempo distortions, however.

Tempo distortions in period quantum measures of the 1st kind are well established and uncontroversial in fertility (Sobotka 2003, 2004a,b; Kohler and Ortega 2002a, 2002b) and in nuptiality (Goldstein 2003; Winkler-Dworak and Engelhardt 2004). Mortality rates of the 1st kind also contain tempo effects, but the period mortality quantum derived from them always equals one because these rates rise with age.

Tempo distortions in period tempo measures of the 1st kind are much less established and we will therefore examine this issue in more detail. The theoretical basis for the existence of such an effect is that tempo measures of the 1st kind are derived from the same numbers of events that produce quantum measures of the 1st kind. As noted earlier, a rising mean age depresses rates of the 1st kind and hence depresses quantum measures calculated from them. When these depressed rates are then used in a life table to obtain a mean age, this mean will contain an upward distortion. Since means of the 2nd kind are not distorted, the difference between the means of the 1st and 2nd kind equals the tempo effect (assuming the constant shape assumption holds).

Tempo effects in the period mean age at first birth

Figures 11-13 compare mean ages at first birth of the 1st and 2nd kind in the Czech Republic, the Netherlands, and Spain. In all three countries these means have risen, but the means of the 1st kind are higher than those of the 2nd kind. The difference between these means is as expected from the operation of the tempo effect. According to the theoretical argument presented earlier, the mean of the 1st kind is distorted because the numerators of rates of the first kind contain tempo effects. Means of the 2nd kind are not distorted because tempo effects in the numerators of rates of the 2nd kind are offset by tempo effects in their denominators. As a result, in years when the mean age at first birth is rising, tempo effects raise the mean of the 1st kind (based on hazard rates) above the mean of the 2nd kind (based on incidence rates). Note that these means are nearly equal to one another in the Czech Republic before 1990 and in the Netherlands after 1997. These are periods when the mean age at first birth did not change and as a result there are no tempo effects. In Spain such a convergence of the two kinds of means is not observed because the mean of the 2nd kind rises throughout the period plotted in Figure 13.

In support of the argument that the mean of the 2nd kind is not distorted, Figures 11-13 include the mean ages at first birth of successive cohorts. The cohort mean age at first birth $M(c)$ for a cohort born in year $c = t - M(c)$ is plotted at time t . This cohort mean age is close to the mean age of the 2nd kind in the Czech Republic and in Spain and falls between the means

of the 1st and 2nd kind in the Netherlands. Theoretical work by Rodriguez (in this volume) and Goldstein (in this volume) has proved that $M(c)$ for people born in year $t - MAE(t)$ equals the mean of the 2nd kind at time t when the translation assumptions hold. Since these assumptions do not hold exactly, $M(c)$ is not exactly equal to the observed mean of the 2nd kind, but the correspondence is good and clearly better than for means of the 1st kind.

Tempo effects in the period mean age at death (i.e., in life expectancy)

Figures 14-16 compare the period mean ages at death of the 1st and 2nd kind for Denmark, England and Wales, and Sweden (as before mortality under age 30 is assumed to be absent). The results are broadly similar to those for first birth: the means have risen over time and the mean of the 1st kind (i.e., period life expectancy) exceeds the mean of the 2nd kind. The difference between the two means again equals the tempo effect. In addition, the lagged cohort mean age at death is close to the mean age of the 2nd kind, which is as expected in a population in which the translation assumptions hold for adult death rates⁸. In sum, our conclusion that the period mean age at a life-cycle event calculated with standard life table methods is distorted by tempo effects is based on and supported by the following findings:

- a. The theoretical analysis of the preceding sections shows that a rising mean age at an event depresses numbers of events in the numerators of period age-specific hazard rates. This inflates the calculated period mean ages calculated from these rates. Similarly, a falling mean age at an event inflates numbers of events in the numerators of period age-specific hazard rates and depresses the mean ages calculated from these rates by standard life table methods.
- b. The observed period mean age based on hazard rates exceeds the period mean age based on incidence rates in populations in which the mean age is rising. Figures 11-13 demonstrate this for first birth and Figures 14-16 for death. This difference is due to a tempo distortion in the mean age based on hazard rates, because, as noted, the mean age based on incidence rates is not affected.
- c. The differences between the period mean ages based on hazard and incidence rates (i.e., the tempo effect) disappear when the mean age stops rising. This is evident for means of the first birth in the Czech Republic before 1990 in Figure 11 and for the Netherlands after 1997 in Figure 12, as well as for the mean ages at death in Denmark in the mid-1990s in Figure 14. These results are of course as expected because when there is no change in the tempo of an event there should be no tempo effect.

⁸ In the calculation of period and cohort tempo of mortality the risk of mortality under age 30 is set to zero, thus insuring the comparability of estimates.

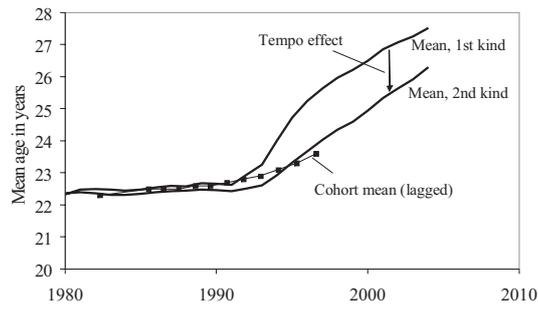


Fig. 11. Mean age at first birth. Means of 1st and 2nd kind and lagged cohort mean, Czech Republic.

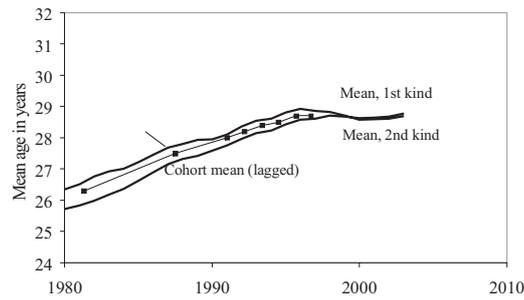
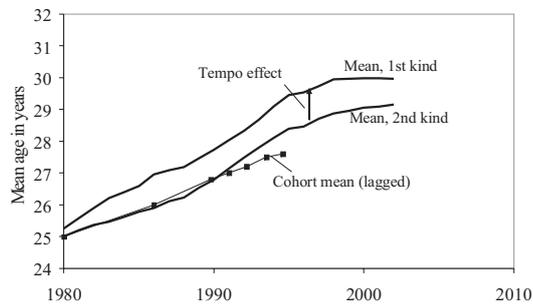


Fig. 12. Mean age at first birth. Means of 1st and 2nd kind and lagged cohort mean, the Netherlands.



Source: Sobotka, 2004a; Sardon 2004b

Fig. 13. Mean age at first birth. Means of 1st and 2nd kind and lagged cohort mean, France.

- d. The cohort mean age for a cohort born in year $c = t - M(c)$ is close to the period mean age of the 2nd kind. This is illustrated in Figures 11-16.

In addition, as shown below, the tempo-adjusted mean of the 1st kind is close to the observed mean of the 2nd kind.

A simple example of the mortality tempo effect

Since a tempo effect in life expectancy is a new and complex concept, we present a simple hypothetical example to demonstrate how the mortality tempo effect operates (Bongaarts and Feeney 2002, in this volume p. 11). Consider a stationary population with a life expectancy at birth of 70 years. Suppose further that a “life extension” pill is invented that defers the death of any person who consumes it by 3 months. If everyone in the population takes this pill on January 1 of year T , there will be no deaths during the first three months of the year. The number of deaths during this year is 25 per cent lower than it would have been without the pill, and the mean age at death is 70.25 years rather than 70 years. Since the pill’s effect is the same at all ages, the level of the force of mortality function is also reduced by 25 per cent, and the age to which each value of the function is attached increases by 0.25 years. This change in the force of mortality function causes life expectancy at birth as conventionally calculated to rise to nearly 73 years for year T (see Figure 17).

In the following year, $T+1$, the number of deaths and the force of mortality function rise to the level observed before year T , but with values shifted forward to older ages by 0.25 years. Life expectancy at birth as conventionally calculated, having risen from 70 years prior to year T to nearly 73 years during year T , falls back to 70.25 years, as shown in Figure 17. This rise and fall in life expectancy at birth as conventionally calculated is a tempo distortion because it is at variance with the known trend in the mean length of life. Distortion of this kind occurs whenever the mean age at death changes.

This illustration demonstrates the operation of the tempo effect that distorts life expectancy under highly simplified hypothetical conditions. The example can be made more realistic in several ways. First, the life-extending pill can be taken year after year from year T onward. In that case, life expectancy will be distorted not only in year T but in every subsequent year as well. The mean age at death will rise over time and the observed life expectancy will continuously exceed the rising mean age at death due to the tempo effect. Second, the strength of the pill can vary from year to year, thus yielding tempo effects that also vary from year to year. Third, continuous change may be approximated by reducing intervals between pill taking while correspondingly reducing the pill’s life-extending effect so that the annual “dose” remains the same. In the limit the mortality pattern in the hypothetical illustration approaches a real adult mortality pattern in a population in which the fixed-shape assumption holds. Subject to this assumption, improvements in adult

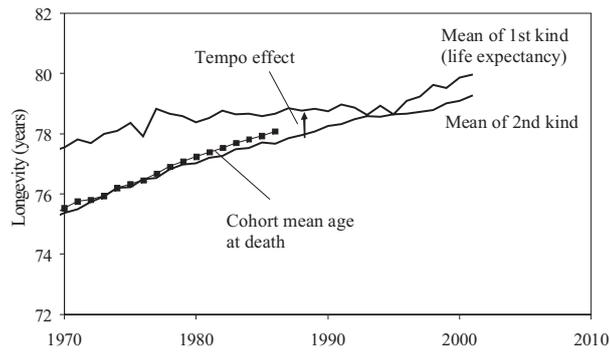


Fig. 14. Mean age at death. Means of the 1st and 2nd kind and lagged cohort mean. Females in Denmark. No mortality under age 30.

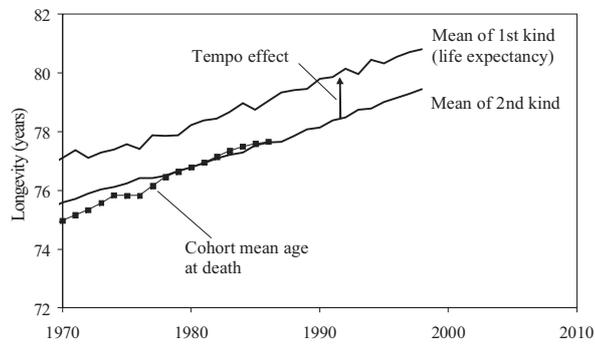
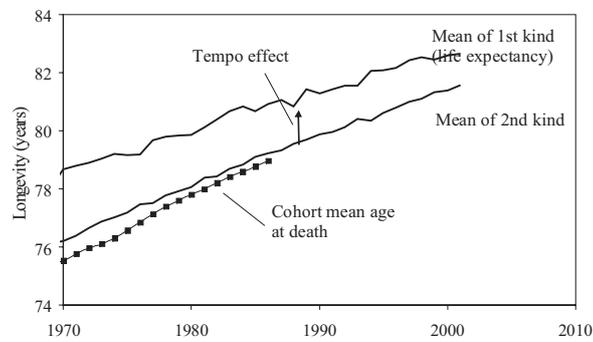


Fig. 15. Mean age at death. Means of the 1st and 2nd kind and lagged cohort mean. Females in England and Wales. No mortality under age 30.



Source: Estimated from data in Human Mortality Database (2005)

Fig. 16. Mean age at death. Means of the 1st and 2nd kind and lagged cohort mean. Females in Sweden. No mortality under age 30.

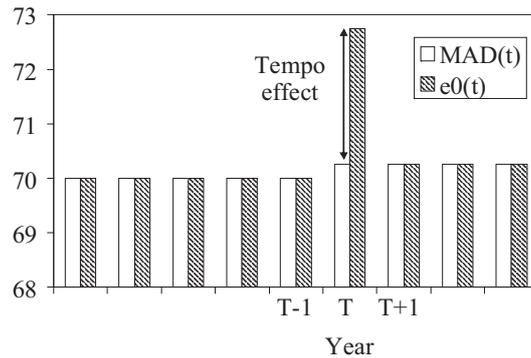


Fig. 17. Hypothetical illustration of effect of increase in mean age at death on conventionally calculated period life expectancy.

mortality can therefore be seen as resulting from the continuous provision of increments to life to all living individuals in every period, with the increments varying over time.

A similar illustration of the impact of a hypothetical “pill” to delay a birth could easily be provided, and it would show a similar tempo distortion of the mean age at birth calculated with a conventional life table.

4.3 Correcting tempo distortions

The method for removing tempo effects from rates of the 1st kind is the same as for rates of the 2nd kind: division of the numerators of the observed rates by the distortion index. Table 6 presents general equations for adjustment as well as applications to first births, first marriages, and deaths. Subject to a constant shape assumption⁹, tempo distortions in summary measures of the 1st kind are removed by dividing the numerators of the hazard rates from which they are derived by $1 - r_p(t)$, where $r_p(t)$ denotes the rate of change in the period mean age of the event.

Observe that, for period measures of the 1st kind, tempo distortions occur for tempo as well as for quantum measures. This is in striking contrast to period measures of the 2nd kind, for which tempo measures are unaffected by tempo distortions if the constant shape assumption holds (because distortions in the numerator and denominator cancel out; see Table 4, right column). For this reason, tempo adjustments are best made using the rate of change in

⁹ In the case of mortality, the constant shape assumption is applied to adult ages (30+) only. Bongaarts and Feeney (in this volume p. 11) demonstrate that for mortality the constant shape assumption is equivalent to assuming that the survival function $p(a, t)$ shifts to higher or lower ages as the mean age at death rises or falls over time and to assuming that the force of mortality $\mu(a, t)$ is proportional to the relative derivative of the survival function with respect to age.

the mean age of the 2nd kind to calculate the distortion index. Note that the procedure used here to make tempo adjustments is different from the one used by Kohler and Ortega (2002a), who rely on the rate of change in the mean age derived from the schedule of rates of the 1st kind. We believe that our approach is more accurate. Note also that if the translation assumption holds, the rates of change calculated from rates of the 1st and 2nd will be equal.

Table 6. Adjustments for tempo distortions in period quantum and tempo measures based on rates of the 1st kind.

Event	Adjusted quantum (total event rate)	Adjusted tempo (mean age at event)
General	$TER_L^*(t) = 1 - e^{-\int_0^\infty \frac{\mu(a,t)}{1-r_p(t)} da}$	$MAE_L^*(t) = \frac{1}{TER_L^*(t)} \int_0^\infty e^{\int_0^a \frac{\mu(x,t)}{1-r_p(t)} dx} da + \int_0^\infty TER_L^*(t) - 1 da$
First birth	$TFR_L^*(t)$	$MAB_L^*(t)$
First marriage	$TNR_L^*(t)$	$MAM_L^*(t)$
Death	$TMR_L^*(t)$	$MAD_L^*(t)$

To illustrate the correction for tempo distortion in tempo measures, we apply the above procedure to mortality, to obtain a tempo-adjusted life expectancy (mean of the 1st kind). It follows that calculated life expectancy at birth may be adjusted for the tempo distortion by dividing the numerators of the observed age-specific death rates by $1 - r_p(t)$ and by using the resulting adjusted age-specific rates in the life table calculation (provided the constant shape assumption holds). This result is equivalent to substituting $TER_L^*(t) = TMR_L^*(t) = 1$ in the tempo equation in the top right cell of Table 6, giving the following tempo-adjusted life expectancy at birth

$$MAD_L^*(t) = e_0^*(t) = \int_0^\infty e^{-\int_0^a \frac{\mu(x,t)}{1-r_p(t)} dx} da, \tag{10}$$

where $r_p(t)$ denotes the rate of change in the period mean age at death $MAD(t)$. Because $1 - r_p(t) = TMR(t)$ (see formula (8) above), (10) may also be written as

$$MAD_L^*(t) = e_0^*(t) = \int_0^\infty e^{-\int_0^a \frac{\mu(x,t)}{TMR(t)} dx} da, \tag{11}$$

which gives more stable results in empirical application. The tempo distortion in the conventional life expectancy at birth equals the difference between $MAD_L(t)$ and $MAD_L^*(t)$.

Bongaarts and Feeney (in this volume p. 11) prove that the tempo-adjusted life expectancy at birth given by (10) or (11) equals the mean age at death calculated from rates of the 2nd kind (i.e., $MAD(t)$ in Table 2),

$$MAD_L^* = MAD(t), \quad (12)$$

provided the constant shape assumption holds.

Table 7 shows empirical estimates for three alternative estimates of the mean age at death (average of annual values for 1970-1990, no mortality under age 30) for females in Denmark, England and Wales, and Sweden¹⁰ :

$MAD(t)$, derived from rates of the 2nd kind (not distorted)
 $MAD_L(t) = e_0(t)$, derived from rates of the 1st kind (distorted)
 $MAD_L^*(t)$, derived from tempo-adjusted rates of the 1st kind (distortion corrected)

These results confirm that $MAD(t)$ and $MAD_L^*(t)$ have nearly the same value as predicted by (12). Table 7 also documents substantial tempo effects in the conventionally calculated life expectancy, $e_0(t) = MAD_L(t)$. The upward distortions in female life expectancy at birth for 1970-1990 are estimated at 1.5 years in Denmark, 1.4 years in England and Wales, and 1.9 years in Sweden. Using an indirect method Bongaarts and Feeney (2002) estimate a distortion of 3.3 years for Japan.

The preceding analysis has demonstrated that tempo-adjusted mortality tempo measures of the 1st and 2nd kind are equal under the constant shape assumption. As shown in Appendix B, this equality holds in general for both tempo-adjusted quantum and tempo measures for any life-cycle event whenever the observed proportion ever having experienced the event, $p(t)$, maintains its shape over time as the mean age at the event rises or falls over time. This condition holds approximately for adult mortality in contemporary low-mortality populations (Bongaarts and Feeney 2002, in this volume p. 11).

5 Conclusion

Demographers have developed a number of widely used methods to estimate the quantum and tempo of life-cycle events. The level of fertility, for example,

¹⁰ The estimates of alternative measures of the mean age at death in Tables 7 and Figure 14-16 assume no tempo effects under age 30. For simplicity, life expectancy at birth is calculated as $e_0 = 30 + e_{30}$ and $e_0^* = 30 + e_{30}^*$, ignoring mortality under age 30. In countries where mortality under age 30 is not small, we recommend the following more general equations for estimating observed and tempo-adjusted life expectancy:

$$e_0 = {}_{30}L_0 + l_{30}e_{30} \quad e_0^* = {}_{30}L_0 + l_{30}e_{30}^*$$

Note also that Bongaarts and Feeney (in this volume p. 11) use yet another way to calculate the period mean age at death, as $\int_0^\infty p(a, t) da$, with $p(a, t)$ denoting the proportion of the cohort born at time $t - a$ who survive to age a . This estimate is identical to the variable called CAL , the cross-sectional average length of life, introduced by Brouard (1986) and Guillot (2003).

Table 7. Alternative estimates of the observed and tempo-adjusted period mean age at death: Average of annual estimates from 1970 to 1990 for females with no mortality under age 30^a.

	Mean age at death, females (average, 1970-1990)			
	$MAD(t)$ (from rates of the 2nd kind)	$MAD_L(t) = e_0(t)$ (from rates of the 1st kind)	$MAD_L^*(t) = e_0^*(t)$ (tempo-adjusted)	Tempo effect $MAD_L(t) -$ $-MAD_L^*(t)$
Denmark	76.8	78.4	76.9	1.5
England and Wales	76.9	78.3	76.8	1.4
Sweden	78.2	80	78.1	1.9

Source: Bongaarts and Feeney 2002, in this volume p. 11. Death rates from University of California, Berkeley Mortality Data Base.

Note: a With no mortality under age 30, $e_0(t) = e_{30}(t) + 30$

is usually measured by the total fertility rate and the level of mortality by the life expectancy at birth. The wide availability, ease of interpretation, and up-to-date nature of these conventional period indicators have led to neglect of some of their deficiencies. Most analysts are aware of inaccuracies due to sampling error and incomplete vital registration, but they often neglect the pervasive influence of tempo distortions of many period indicators of life-cycle events.

Tempo distortions in period fertility measures were discovered more than half a century ago and are generally acknowledged. The post-war baby boom in the United States, for example, was due in part to a decline in the age at childbearing, and the recent low total fertility rates in many developed countries are in part due to delays in childbearing. This study argues that similar tempo distortions can occur in period measures of other life-cycle events, including marriage and death. This is the case even for measures derived from period life tables such as life expectancy at birth. These distortions are not generally recognised and are rarely if ever taken account of in empirical analysis.

Comparisons of period and cohort measures indicate that tempo distortions can be substantial in size. Distortions in the total fertility, marriage, and mortality rates of more than 10% were common during the 1980s. Using distorted age-specific death rates in a mortality life table leads to distorted estimates of life expectancy (typically exaggerated by 1-2 years).

The adjustment method proposed earlier by Bongaarts and Feeney is shown both by theoretical argument and by empirical example to be an effective, if approximate, solution to the problem of adjusting tempo and quantum measures for life-cycle events. Although this approach makes a simplifying assumption about changes over time in the age patterns of event rates, the results appear generally robust to deviations from this assumption.

The adjusted period tempo and quantum measures should be interpreted as variants of their conventional counterparts. The total fertility rate, for

example, is defined as the average number of births for a hypothetical cohort of women subjected throughout life to the age-specific birth rates observed in a given year. This is a hypothetical rate because no actual cohort will experience these observed period birth rates. The tempo-adjusted total fertility rate is a similarly hypothetical measure, but one that corrects for distortions caused by year-to-year tempo changes. Neither the observed nor the adjusted total fertility rate attempts to estimate the fertility rate of any actual cohort, nor do they attempt any prediction of future fertility. The goal of the tempo adjustment is simply to provide period quantum and tempo measures that are free of the tempo distortions in conventional measures.

Adjusted period measures are hypothetical in that they tell us what the observed period measure would have been if there had been no change in the timing of the event. This hypothetical measure does not correspond to the behaviour of a particular cohort because the translation assumptions are often violated. However, as we have seen in this chapter, the patterns of change observed in practice are often close enough to the translation assumptions that the adjusted period measures are approximate measures of lagged cohorts. Furthermore, the adjusted period measures give us an indication of what rates could be like in the future if postponement comes to an end.

Distorted views of past levels and trends in the quantum and tempo of life-cycle events may lead to misleading projections and to the adoption of sub-optimal social and health policies. It is therefore desirable for analysts to understand the strengths and weaknesses of period indicators of life-cycle events and to recognise and correct tempo distortions.

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Appendix A: Relationship between the completed fertility rate and the weighted average of tempo-adjusted period total fertility rates

Let age-specific fertility rates at time t and age a be denoted $d(a, t)$. The total fertility rate equals

$$TFR(t) = \int d(a, t) da \quad (1a)$$

The distribution of fertility by age at time t is denoted $f(a, t)$:

$$f(a, t) = \frac{d(a, t)}{TFR(t)} \quad (2a)$$

so that $\int f(a, t) da = 1$ and $d(a, t) = TFR(t)f(a, t)$.

The completed fertility rate for the cohort born in year t_0 equals

$$CFR(t_0) = \int d(a, t_0 + a) da = \int TFR(t_0 + a)f(a, t_0 + a) da \quad (3a)$$

rearranging (6) yields

$$TFR(t) = [1 - r_p(t)]TFR^*(t) \quad (4a)$$

and substitution of (4a) in (3a) gives

$$\begin{aligned} CFR(t_0) &= \int TFR^*(t_0 + a)[1 - r_p(t_0 + a)]f(a, t_0 + a) da \\ &= \int TFR^*(t_0 + a)\nu(a, t_0) da \end{aligned} \quad (5a)$$

where $\nu(a, t_0) = [1 - r_p(t_0 + a)]f(a, t_0 + a)$.

The weighted average of $TFR^*(t)$ is defined as

$$\overline{TFR}(t_0) = \frac{\int TFR^*(t_0 + a)\nu(a, t_0) da}{\int \nu(a, t_0) da} = \int TFR^*(t_0 + a)\omega(a, t_0) da \quad (6a)$$

where $\omega(a, t_0) = \frac{\nu(a, t_0)}{\int \nu(a, t_0) da}$
 It follows from (5a) and (6a) that

$$CFR(t_0) = \overline{TFR}(t_0) \int \nu(a, t_0) da \quad (7a)$$

Equations (5a), (6a), and (7a) hold in general and do not require any simplifying assumptions. However, it can be shown that $\int \nu(a, t_0) da = 1$ and $\omega(a, t_0) = \nu(a, t_0)$ when the constant shape assumption holds. In that case $CFR(t_0) = \overline{TFR}(t_0)$.

Appendix B: Comparison of measures of the 1st and 2nd kind

If age-specific rates change without conditions, then period quantum and tempo measures of the 1st kind generally differ from measures of the 2nd kind. We will now demonstrate that this difference between measures of the 1st and 2nd kind disappears if the tempo effect is removed and if the shape of the proportion ever having experienced the event remains invariant as the mean age at the event changes. Holding the shape of $p(t)$ constant implies

$$p(a, t) = p(a - S(t), 0) \text{ for } a \geq S(t) \text{ and } p(a, t) = 1 \text{ for } a < S(t) \quad (1b)$$

where $S(t)$ is equal to the amount of the shift since $t = 0$. As shown by Bongaarts and Feeney (2002, in this volume p. 11) (1b) implies that

$$d(a, t) = [1 - r_p(t)] \frac{-\partial p(a, t)}{\partial a} \quad (2b)$$

and

$$\mu(a, t) = [1 - r_p(t)] \frac{\frac{-\partial p(a, t)}{\partial a}}{p(a, t)} \quad (3b)$$

Let the tempo-adjusted versions of $\mu(a, t)$ and $d(a, t)$ be denoted $\mu^*(a, t)$ and $d^*(a, t)$ respectively; then

$$d^*(a, t) = \frac{d(a, t)}{1 - r_p(t)} = \frac{-\partial p(a, t)}{\partial a} \quad (4b)$$

and

$$\mu^*(a, t) = \frac{d^*(a, t)}{p(a, t)} = \frac{\frac{-\partial p(a, t)}{\partial a}}{p(a, t)} \quad (5b)$$

It follows from (4b) and (5b) that

$$p(a, t) = 1 - \int_0^a d^*(x, t) dx = 1 - e^{-\int_0^a \mu^*(x, t) dx} \quad (6b)$$

Rearranging (6b) and integrating to m , the highest age at which the event is observed, gives

$$e^{-\int_0^m \mu^*(x, t) dx} p(a) = 1 - p(m, t) \quad (7b)$$

and

$$\int_0^m d^*(x) dx = 1 - p(m, t) \quad (8b)$$

Substitution of (7b) and (8b) in the equations for tempo-adjusted quantum of the 1st and 2nd kind (from Tables 4 and 6 respectively) shows that the tempo-adjusted quantum of the 1st kind

$$TER_L^*(t) = 1 - e^{-\int_0^m \frac{\mu(a, t)}{1-r_p(t)} da} = 1 - e^{-\int_0^m \mu^*(a, t) da} = 1 - p(m, t) \quad (9b)$$

equals the tempo-adjusted quantum of the 2nd kind

$$TER_L^*(t) = \int_0^m \frac{d(a, t)}{1-r_p(t)} da = \int_0^m d^*(a, t) da = 1 - p(m, t) \quad (10b)$$

Similarly, the tempo-adjusted mean age of the 1st kind

$$\begin{aligned} MAE_L^*(t) &= \frac{1}{TER_L^*(t)} \int_0^m e^{-\int_0^a \frac{\mu(x, t)}{1-r_p(t)} dx} + TER_L^*(t) - 1 da \\ &= \frac{1}{1-p(m, t)} \int_0^m e^{-\int_0^a \mu^*(x, t) da} - p(m, t) da \\ &= \frac{1}{1-p(m, t)} \int_0^m p(a, t) - p(m, t) da \end{aligned} \quad (11b)$$

equals the tempo-adjusted mean age of the second kind

$$\begin{aligned} MAE^*(t) &= \frac{1}{TER_L^*(t)} \int_0^m \frac{ad(a, t)}{1-r_p(t)} da \\ &= \frac{1}{1-p(m, t)} \int_0^m ad^*(a, t) da \\ &= \frac{1}{1-p(m, t)} \int_0^m a \frac{\partial p(a, t)}{\partial a} da \\ &= \frac{1}{1-p(m, t)} \int_0^m p(a, t) - p(m, t) da \end{aligned} \quad (12b)$$

Whenever (1b) holds, tempo-adjusted quantum and tempo measures of the 1st and 2nd kind are equal to one another.

The finding that $TER_L^*(t) = TER^*(t)$ and $MAE_L^*(t) = MAE^*(t)$ is of interest because it implies that quantum and tempo measures of the first kind are equal to those of second kind even when the age-specific proportions of individuals exposed to the risk of an event are changing, provided (1b) holds. Bongaarts and Feeney (in this volume p. 11) have examined this assumption for mortality and found that it provides a good approximation of reality in recent decades among adults in countries with low mortality. But the assumption is probably less applicable to the fertility and nuptiality processes in which quantum changes can occur at the same time as tempo changes. Further research is required on the implication of deviations from this assumption.