MAX PLANCK INSTITUTE
FOR DEMOGRAPHIC RESEARCH

Konrad-Zuse-Strasse 1 · D-18057 Rostock · Germany · Tel +49 (0) 3 81 20 81 - 0 · Fax +49 (0) 3 81 20 81 - 202 · www.demogr.mpg.de

MPIDR Working Paper WP 2023-032 I August 2023
https://doi.org/10.4054/MPIDR-WP-2023-032

Decomposition of differences between life expectancy losses or gains: relative change and absolute level components. A Research Note

Vladimir M. Shkolnikov I shkolnikov@demogr.mpg.de
Dmitry A. Jdanov I jdanov@demogr.mpg.de
David A. Leon

Decomposition of differences between life expectancy losses or gains: relative change and absolute level components. A Research Note

Vladimir M. Shkolnikov[I], Dmitry A. Jdanov[I] and David A. Leon[II]

[I] Max Planck Institute for Demographic Research, Rostock, Germany

[II] London School of Hygiene and Tropical Medicine

Abstract

When comparing life expectancy gains or losses between populations separating the effects of relative mortality changes from those due to differences in absolute mortality levels may be of interest. We propose a simple procedure for calculating these Change and Level components of life expectancy losses or gains in a target population compared to a reference population. Two empirical examples illustrate the use of the Change-Level decompositions in an analysis of differences between life expectancy losses due to the COVID-19 pandemic in 2021.

Key words. Life expectancy, temporal changes, inter-population differences, decomposition, additive contributions, East-West mortality divide, COVID-19 pandemic

Introduction

The same relative change in age-specific death rates generates a smaller or larger life expectancy impact depending on the absolute baseline level of death rates. When comparing life expectancy dynamics between populations, analysts might be interested in accounting for this effect.

This research note presents an approach to quantifying the separate impacts of relative mortality changes and differences in absolute levels of mortality on comparative dynamics of life expectancy. Such quantification is particularly sensible in analyses of health shocks when many populations with different initial levels of mortality face simultaneously strong exposure to a health hazard of the same nature. This was the case in 2020-2021 when many countries experienced abrupt life expectancy losses due to the COVID-19 pandemic. It may be useful also in comparative analyses of life expectancy in sub-populations with persistently different levels of mortality (e.g. males vs. females, upper vs. lower socioeconomic group, etc.).

We propose a simple decomposition procedure to resolve inter-population differences in life expectancy changes into relative Change and absolute Level components.

Method

A difference in life expectancy change between two populations may be considered as dependent on A) the difference between the populations in the relative mortality changes (Change component); B) the difference between the populations in the absolute baseline mortality levels (Level component). In the substantive sense, the former component reflects an impact of level-independent improvement or deterioration of population health, and the latter component reflects an impact of the baseline level of population health.

A change in the life expectancy at birth in a population can be expressed as a difference between the baseline and the new values of the life expectancy at birth:

$$\Delta = e_0(\boldsymbol{M}^b) - e_0(\boldsymbol{M}) = e_0(\boldsymbol{M}^b) - e_0(\boldsymbol{k}\circ\boldsymbol{M}^b). \qquad (1)[1]$$

In Eq. (1), the life expectancy at birth $e_0(\cdot)$ is presented as a function of the vector of age-specific death rates $\boldsymbol{M} = (M_1, M_2, \dots M_n)$; M and $M^{\,b}$ are the new and the old (baseline) vectors of age-specific death rates M_i. The new vector of age-specific death rates may be presented as an elementwise (Hadamard) product $\boldsymbol{k}\circ\boldsymbol{M}^b = (k_1 \cdot M_1^b, k_2 \cdot M_2^b, \dots, k_n \cdot M_n^b)$ with coefficients k_i expressing the *relative mortality change*.

Let us consider two populations: a *target* population and a *reference* population. Given Eq. (1), the life expectancy changes in these populations are:

$$\Delta_{TGT} = e_0(\boldsymbol{M}_T^b) - e_0(\boldsymbol{k}_T\circ\boldsymbol{M}_T^b)\,, \qquad (2a)$$

$$\Delta_{REF} = e_0(\boldsymbol{M}_R^b) - e_0(\boldsymbol{k}_R\circ\boldsymbol{M}_R^b)\,. \qquad (2b)$$

The target-reference gap in the life expectancy losses is $\Delta_{T-R} = \Delta_{TGT} - \Delta_{REF}$. This gap depends on A) the C*hange* – the target-reference difference between elements of the vectors $\boldsymbol{k}_T$ and $\boldsymbol{k}_R$; and B) the *Level* – the target-Reference difference between elements of the vectors $\boldsymbol{M}_T^b$ and $\boldsymbol{M}_R^b$.

To estimate the Change and the Level components of the total difference Δ_{T-R} , we calculate first a counterfactual life expectancy change that would be observed in the target population if the relative mortality increase in this population was equal to the relative mortality increase in the reference population:

$$\Delta_{Cf} = e_0(\boldsymbol{M}_T^b) - e_0(\boldsymbol{k}_R\circ\boldsymbol{M}_T^b)\,. \qquad (3)$$

Eq. (3) was obtained by replacing $\boldsymbol{k}_T$ by $\boldsymbol{k}_R$ in Eq. (2a).
Then the Change component Δ_C and the Level component Δ_L may be expressed as

[1] In this and the following three equations, it is equally possible to define the life expectancy change as the new life expectancy value minus the baseline life expectancy value: $\Delta = e_0(\boldsymbol{M}) - e_0(\boldsymbol{M}^b) = e_0(\boldsymbol{k}\circ\boldsymbol{M}^b) - e_0(\boldsymbol{M}^b)$.

$$\Delta_C = \Delta_{TGT} - \Delta_{Cf} \qquad (4a)$$
$$\Delta_L = \Delta_{Cf} - \Delta_{REF} \qquad (4b)$$

Equations (2a) to (4b) enable us to present the target-reference difference between the life expectancy changes as a sum of the Change and the Level components

$$\Delta_{T-R} = \Delta_C + \Delta_L. \qquad (5)$$

The following two sections provide empirical examples of decompositions in an analysis of life expectancy losses in 2021 caused by the COVID-19 pandemic.

Example 1. East-West differences in life expectancy losses in 2021: Slovakia vs. England and Wales

The long-lasting East-West gap in life expectancy between Eastern (former communist bloc) countries and Western (other European countries) further widened in 2020-2021 due to the COVID-19 pandemic. The Eastern life expectancy losses (compared to the Lee-Carter predicted life expectancies) were especially large in 2021 (Schöley et al. 2022). In 2021 all countries of the former communist bloc except Slovenia experienced distinctly higher life expectancy losses than Western European countries.
Figure 1 compares the relative mortality excess and the baseline mortality across ages in 2021 between Slovakia (Eastern) and England and Wales (Western). The upper panels show the relative excess in death rates in the two populations in 2021 for males and females. In both countries, the relative excess does not show any systematic association with age. In the age range of 30-70 years, the relative excess was higher in Slovakia compared to England and Wales with a mean difference of 19% and 21% for males and females, respectively.
The lower panels of Figure 1 show the baseline death rates which as expected show steep exponential increases with age in both countries, but at any age higher in Slovakia than in England and Wales. This gap in the level was substantially wider for males than females with the maximal (about two-fold) difference seen among males aged 45-49 to 65-69 years. Figure 1 also shows that despite the large gender difference in absolute mortality, the relative mortality excess was quite similar for men and women in both countries. In fact, in Slovakia, the relative excess was even slightly higher for women than for men.
In 2021, the baseline (Lee-Carter predicted[2]) values of the life expectancy at birth in Slovakia (target) and England and Wales (reference) were: 74.80 and 80.19 years for males and 81.41 and 83.78 years for females, respectively. The observed life expectancies in Slovakia and England and Wales in 2021 were: 71.30 and 78.70 years for males and 78.22 and 71.30 years for females, respectively. In terms of life expectancy losses in 2021 for males, these were 3.50 years in Slovakia and 1.48 years in England and Wales, with the corresponding female life

[2] See Islam et al. (2022) for detailed explanation of method for assessment of the COVID-19 related life expectancy losses.

expectancy losses being 3.09 and 1.05 years. Thus, the differences in life expectancy losses between Slovakia and England and Wales in 2021 were 2.02 and 2.04 years for males and females, respectively.
Table 1 results of the decomposition of the target-reference differences in life expectancy losses according to Eq. (1) to Eq. (5) are treating Slovakia as the target population and England as the reference population (see also Supplementary file Example1.xlsx for the underlying calculations).
The results suggest that 82% (1.65/2.02) of the overall difference for males was due to a higher relative mortality increase in Slovakia compared to England and Wales, while for females the equivalent was 96% (1.96/2.04). In other terms, while for females only 4% of the difference in life expectancy changes was contributed by differences in baseline mortality between the countries for males it was more substantial at 18%.

Example 2. Gender differences in life expectancy losses in 2021: males vs. females in England and Wales and Slovakia

In this example, our interest is in understanding sex differences in changes in life expectancy in the two countries. In 2021, the difference between the male (target) and female (reference) losses constituted 0.41 years (3.50 minus 3.09 years) in Slovakia and 0.43 years (1.48 minus 1.05 years) in England and Wales.
Table 2 shows the results of the decompositions of the gender differences within the two populations (see Supplementary file Example2.xlsx for calculations). In England and Wales, the Change and the Level components of the gender difference were quite close to each other: 0.19 years (43.2%) and 0.24 years (56.8%), respectively. In Slovakia, the total gender difference in the losses of 0.41 years was a result of a balance between the positive Level component (0.75 years) which outweighed the negative Change component (-0.34 years).

Discussion

When comparing the extent of changes in life expectancy between populations it is useful to to distinguish contributions to these changes that result from differences in relative mortality from those due to those due to differences in absolute baseline mortality levels. We proposed a simple procedure for the calculation of the corresponding two components. It enables an analyst to account for unequal baseline levels of mortality when comparing life expectancy losses or gains between populations with substantially different levels of mortality.
While the conventional decomposition methods (Andreev 1982; Pollard 1982; Arriaga 1984; Pressat 1985; Andreev et al. 2002; Vaupel and Canudas-Romo 2003; Shkolnikov et al. 2006; Horiuchi et al. 2008; Beltran-Sanchez et al. 2008) aim at decomposing life expectancy change within a single population or life expectancy difference between two populations, the proposed procedure aims at decomposing a difference between life expectancy changes in a target population compared to a reference population.

The two empirical examples highlighted the variable importance of the Change and the Level components. In the first example, we compared life expectancy losses between Slovakia, an Eastern European country with a high (by European standard) level of baseline mortality (especially among males) and with one of the highest relative mortality excesses in 2021 to England and Wales representing West of Europe concerning both absolute level and relative change. A higher baseline Level could be expected to contribute to the larger life expectancy losses in Slovakia compared to England and Wales. We found, however, that the corresponding contribution was moderate for men and almost negligible for women. The much higher life expectancy loss in 2021 in Slovakia compared to England and Wales was mostly produced by the relative mortality rise (the Change component). A simple interpretation of this result is that only a small fraction of the pandemic-induced differences in life expectancy in Slovakia compared to England and Wales can be explained by the poorer health and survival of the Slovakian population. Instead, higher levels of exposure to the COVID-19 virus and lower levels of immunity are likely to underlie the bulk of this difference.

The male-female difference in life expectancy losses within Slovakia and England and Wales could be expected to be reinforced by the Level component because of the substantially lower level of female death rates across the entire range of ages in each country. And indeed, in each of the two countries, the gender gap in the losses was found to be largely (England and Wales) or solely (Slovakia) determined by the Level component[3]. It suggests a smaller gender difference in exposure to the virus but a higher contribution of poorer baseline health among males.

In this research note our focus has been exclusively on life expectancy at birth. It is also possible to apply the same procedure to life expectancies at other ages (e_{15}, e_{30}, e_{50}, e_{65}, etc.). Moreover, the method might be applied not only to life expectancy but also to other aggregate demographic or public health measures.

The Change-Level decomposition has limitations. First, it would not be useful to use it when the target-reference difference in the baseline mortality are small at all or most ages. Second, our approach focuses on disentangling the effects of relative changes from those of levels of absolute baseline differences. However, it cannot be directly extended to simultaneously partition out the effects of different age patterns of mortality. Unlike the conventional stepwise replacement algorithm (Andreev et al. 2002), we replace entire vectors rather than run a sequential replacement of age-specific elements of these vectors. If one wished to conduct a complementary split by age, a more complex approach would be needed along the lines of the contour decomposition method (Jdanov et al. 2017).

---

[3] An example of similar but numerically greater impact of the Level was observed in the 1990s. The exceptional life expectancy fall in Russia in the early 1990s was twice larger among males than among females despite almost equal relative increases in male and female death rates (Leon et al. 1997). This seemingly illogical result was explained by a much higher level of mortality among males than females just before the health crisis.

Supplementary data

The supplementary Excel files provide numerical data used in Figure 1 as well as data and calculations for Tables 1 and 2 (according to Equations (1) to (5)).

References

Andreev, E.M. (1982). *Metod komponent v analize prodoljitelnosty zjizni*. [The method of components in the analysis of length of life]. *Vestnik Statistiki*, 9, 42-47. Available at

Andreev, E.M., Shkolnikov, V.M., and Begun, A.Z. (2002). Algorithm for decomposition of differences between aggregate demographic measures and its application to life expectancies, healthy life expectancies, parity-progression ratios and total fertility rates. *Demographic Research* 7(14): 499–522. http://doi:10.4054/DemRes.2002.7.14

Arriaga, E. (1984). Measuring and explaining the change in life expectancies. *Demography* 21(1): 83–96. http://doi:10.2307/2061029

Beltran-Sanchez, H., Preston, S., and Canudas-Romo, V. (2008). An integrated approach to cause-of-death analysis: cause-deleted life tables and decompositions of life expectancy. *Demographic Research* 19(35): 1323–1350. http://doi:10.4054/DemRes.2008.19.35

Horiuchi, S., Wilmoth, J., and Pletcher, S. (2008). A decomposition method based on a model of continuous change. *Demography* 45(4): 785–801. http://doi:10.1353/dem.0.0033

Islam, N., Jdanov, D.A., Shkolnikov, V.M., Khunti, K., Kawachi, I., White, M., Lewington, S., Lacey, B. (2021) Effects of covid-19 pandemic on life expectancy and premature mortality in 2020: time series analysis in 37 countries. *BMJ* 375:e066768. http://dx.doi.org/10.1136/bmj-2021-066768

Jdanov, D.A., Shkolnikov, V.M., van Raalte, A., Andreev, E.M. (2017). Decomposing current mortality differences into initial differences and differences in trends: the contour decomposition method. *Demography* 54: 1579-1602. https://doi.org/10.1007/s13524-017-0599-6

Leon, D.A., Chenet, L., Shkolnikov, V., Zakharov, S. et al. (1997). Huge variation in Russian mortality rates 1984-1994. Artefact, alcohol or what? *Lancet*, vol. 350: 383-388. https://doi.org/10.1016/S0140-6736(97)03360-6

Pollard, J.H. (1982). The expectation of life and its relationship to mortality. *Journal of the Institute of Actuaries* 109: 225–240. http://doi:10.1017/S0020268100036258

Pressat, R. (1985). Contribution des écarts de mortalité par âge à la différence des vies moyennes. *Population* 40(4-5): 766–770. http://doi:10.2307/1532986

Schöley, J., Aburto, J.M., Kashnitsky, I. et al. (2022) Life expectancy changes since COVID-19. *Nat Hum Behav* 6, 1649–1659. https://doi.org/10.1038/s41562-022-01450-3

Shkolnikov, V.M., Andreev, E.M., Jasilionis, D., Leinsalu, M., Antonova, O.I., McKee, M. (2006). The changing relation between education and life expectancy in Central and Eastern Europe in the 1990s. *Journal of Epidemiology and Community Health*, 60: 875-881. doi: https://doi.org/10.1136/jech.2005.044719

Vaupel, J. and Canudas-Romo, V. (2003). Decomposing change in life expectancy: A bouquet of formulas in honor of Nathan Keyfitz's 90th birthday. *Demography* 40(2): 201–216. https://doi:10.1353/dem.2003.0018

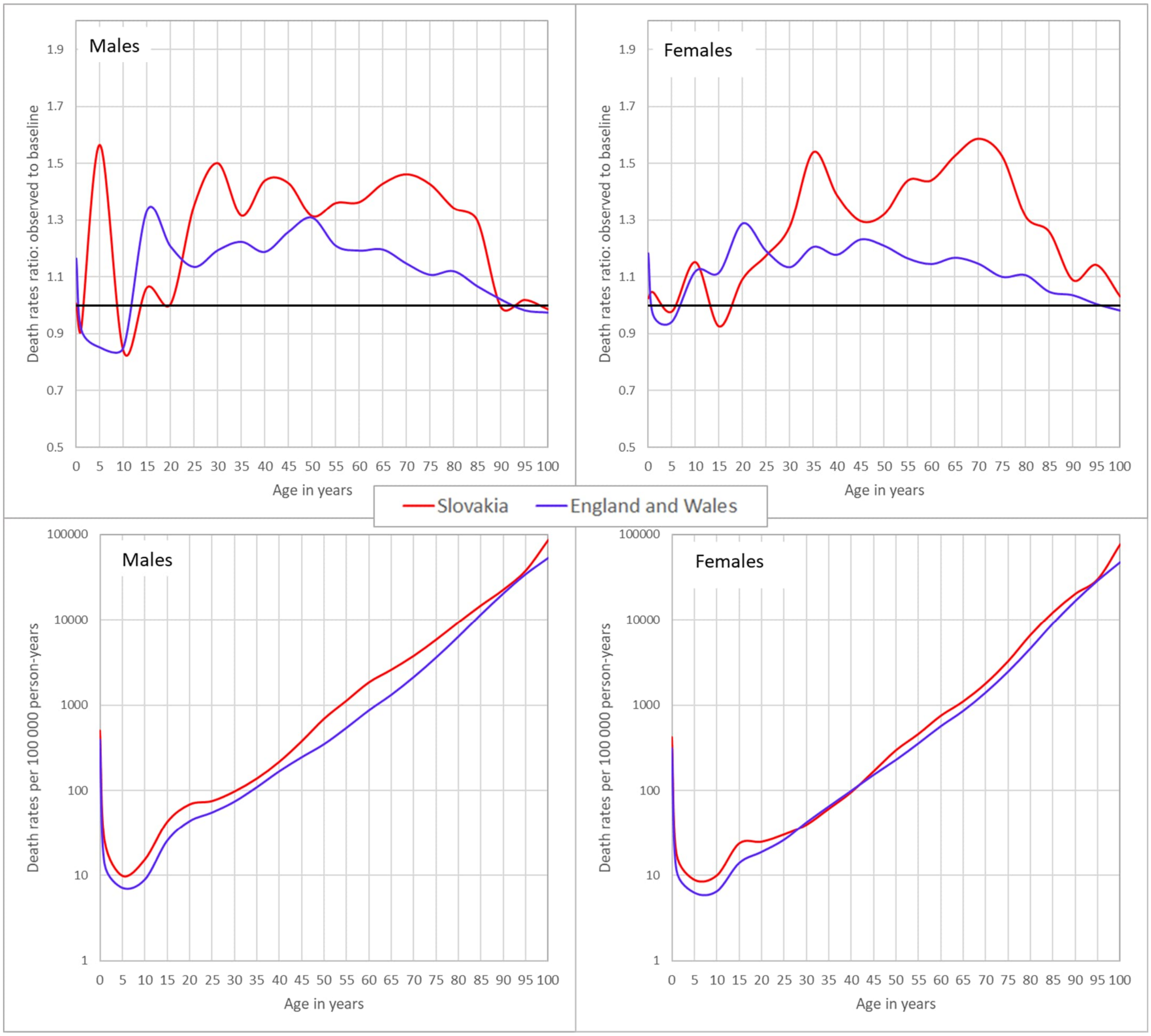

Figure 1. Relative excess in death rates (upper panels) and baseline death rates (lower panels) for Slovakia and England and Wales in 2021

Note. Data and calculations used in this table are given in the supplementary file Figure1.xlsx

Table 1. Differences in life expectancy losses between Slovakia and England and Wales in 2021 and their components produced by relative mortality excess and by baseline mortality

Measure			Males	Females
LE losses observed in Slovakia (target)	Δ_{TGT}	(a)	3.50	3.09
LE losses observed in England and Wales (reference)	Δ_{REF}	(b)	1.48	1.05
Difference (target – reference)	Δ_{T-R}	(a)-(b)	2.02	2.04
LE losses counterfactual in Slovakia (target)	Δ_{Cf}	(c)	1.85	1.14
LE losses due to relative mortality excess (“Change component”)	Δ_{C}	(a)-(c)	1.65	1.96
LE losses due to higher baseline mortality (“Level component”)	Δ_{L}	(c)-(b)	0.37	0.08

Note. Data and calculations used in this table are given in the supplementary file Example1.xlsx.
Equations and explanations concerning the „Deltas” are given in the Methods

Table 2. Gender differences in life expectancy losses in England and Wales and Slovakia in 2021 and their components produced by relative mortality excess and by baseline mortality

Measure			England and Wales	Slovakia
LE losses observed, males (target)	Δ_{TGT}	(a)	1.48	3.50
LE losses observed, females (reference)	Δ_{REF}	(b)	1.05	3.09
Difference (target-reference)	Δ_{T-R}	(a)-(b)	0.43	0.41
LE losses counterfactual for males (target)	Δ_{Cf}	(c)	1.30	3.85
LE losses due to relative mortality excess (“Change component”)	Δ_{C}	(a)-(c)	0.19	-0.34
LE losses due to higher baseline mortality (“Level component”)	Δ_{L}	(c)-(b)	0.24	0.75

Note. Data and calculations used in this table are given in the supplementary file Example2.xlsx.
Equations and explanations concerning the “Deltas” are given in the Methods